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The valuation of industrial property with declining operation returns

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THE VALUATION OF INDUSTRIAL PROPERTY WITH DECLINING OPERATION RETURNS

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The valuation of industrial property with declining operation returns

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Il Geon Yoo

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TABLE OF CONTENTS

I. INTRODUCTION

The valuation and appraisal of the industrial properties are necessary for many reasons [1]:

1. to provide information for management,
2. for tax assessment,
3. for sale or transfer of a business,
4. for condemnation,
5. for settling estates,
6. to set insurance rates, and
7. in issuing securities and financing purposes.

The problem or situation covered by this study came from the appraisal of industrial property and equipment especially for assessing taxes for ad valorem tax purposes. In Iowa and some other states, ad valorem taxes are based on the market value of a property. However, in the majority of situations, market values of the properties cannot be ascertained directly. This may be due to the uniqueness of a property or due to the rarity or nonexistence of market exchanges of comparable property. Thus, determining a procedure to estimate the value of such industrial items or groups of properties is desireable.

Since there are many interpretations of the words value and valuation, it is difficult to use the terms without defining them for estimating the value of industrial properties. Marston et al. wrote about value [1]

> Value is a relative term by which the desirability of ownership of the property in question is stated in terms of other property or money.

For more specific concepts of value, Marston et al. and Bonbright explained into two basic concepts, which are the market value and value to the owner. For market value, Bonbright explained [2]:

> The valuation of property under market value means merely an attempt to estimate the price for which the property could be sold by some stipulated seller to anyone else the conditions of the assumed sale being left for selection by reference to the purpose for which the valuation is being made.

Bonbright also called market value an exchange value. This arose from the definition of market value as the power a commodity commands in exchange of itself for other things. Another definition of the market value is [3]:

> The amount at which property would exchange between a willing buyer and a willing seller, each having a reasonable knowledge of all pertinent facts, neither being under compulsion to buy or sell and with equity to both.

As one of the basic concepts of value, the value to the owner was illustrated by Bonbright [2]:

> The value to the owner is a subjective value representing a state of mind, an attitude of the owner towards the thing valued. The value of a property to its owner is identical in amount with the adverse value of the entire loss, direct and indirect, if he were to be deprived of the property.

There are three different ways of interpreting value to the owner: 1) what the owner actually feels, 2) the results of an objective appraisal, and 3) hindsight evaluation. Thus, the value to the owner can be defined as the worth of the property to the owner himself, and thus, a subjective value. On the contrary, the market value is the price for which the specific property could actually be sold in the open market of many buyers, and hence, an objective value.

A. Evidences of Value

It is usually impossible to measure the value of a property directly. Thus, the appraisal process depends upon judgment based on various evidences of value. The valuation procedure is usually divided into four categories [2]: 1) define purposes of the valuation, the parties, place, and time, 2) define property to be considered, 3) develop the evidences, and 4) weigh the evidences and determine estimate of value.

From the above, the evidences of value are of interest and require further explanation. Marston et al. presented three evidences of value in their text [1]:

> Market price, cost of replacing the service rendered by the property, and present value of the future returns from the property are usually relatively good measures of the value of property to the owner.

Babcock divided the category of evidences as the income method, the replacement cost method, and the market comparison method [4].

Based on the sources cited above, three evidences of value can be defined. These are: 1) market evidence, 2) cost evidence, and 3) income evidence. These three approaches are commonly accepted as bases for judgmental determination of value. A description of each of these evidences follows.

B. The Market Evidence

The market evidence of value consists of an investigation of prices paid for similar properties in a ready and open market. The market evidence usually provides the strongest indication of value among the three evidences, owing to the advantage of comparability with adjacent or similar items sold before. Bonbright supported this when he stated an opening to his market evidence discussion as follows:

> The method of valuation which will now be discussed is given first place, sometimes to the exclusion of all other evidences, in the legal valuation of marketable forms of property [2].

Even though the market evidence is considered to be a strong evidence of value, many kinds of properties do not enjoy a readily marketable status. Again, quoting Bonbright:

> But only with respect to highly marketable property, and not always even there, is a court or appraiser justified in accepting uncritically the record of current sales as the measure of market value [2].

The market evidence, therefore, is considered to be a strong evidence of value if the property has been frequently traded on the open market.

Therefore, with the support of a lot of relevant sales evidences in the market, the market evidence can be most valuable when market value is the objective.

C. The Income Evidence

Income evidence is based on the economic concept that the present value of any object of wealth is simply the discounted value of the anticipated benefits derivable by the owner. That is, the income approach entails an estimate of the net monetary benefits accruing to the prospective buyer of the property every year, and an estimate of the number of years that such benefits to their worth at the present time.

For this purpose, value engineers use a discount rate equal to the rate of return that the prospective buyer would consider reasonable and acceptable for an investment in the property.

Procedure for this income approach is in two steps:

1. Estimation of value of net benefits (income) and future date of realization.
2. Use appropriate rate of discount considering pure interest and risk factor.

However, the income evidence sometimes faces the difficulty in the estimation of accurate future income, discount rate, and related data.

D. The Cost Evidence

The basis of the cost evidence is that value is evidenced by the cost of the property adjusted for service consumed. Four kinds of cost have been identified. These are 1) the original cost, 2) the trended original cost, 3) the reproduction cost, and 4) the replacement cost. The original cost basis is not frequently used because of the effect of the passage of time on the monetary standard used to measure value.

Thus, the trended original cost, reproduction cost, and replacement cost are mostly used. Marston et al. defined them [1]:

1. Trended original cost: original cost converted to current cost by use of cost indexes.
2. Reproduction cost: the estimated cost of reproducing substantially the identical property at a price level as of the date specified.
3. Replacement cost: the estimated cost of replacing the service

of the existing property by another property to achieve the most economical and preferred service, but at prices as of the date specified.

In the estimation of the cost evidence, an appropriate adjustment on deduction is made from the estimated cost new, in an amount representing the loss of value in the property when compared to similar property in new condition.

The loss of value is broadly termed depreciation. The factors making up this loss in value include:

1. reduced service life expectancy,
2. physical deterioration,
3. functional obsolescence, and
4. economic obsolescence.

The cost evidence approach is frequently used for industrial properties since the required information is usually available.

E. Application of Three Evidences

The three approaches for developing value evidence which are commonly accepted were presented in the previous section. In the valuation of industrial property, however, these three evidences are not always applicable. Market evidence is sometimes unavailable due to the lack of sufficient sales data for relevant properties. Few transactions that have taken place often reflect only a scrap value received for the property. Similarly, the income evidence is often hard to obtain because each equipment unit may be only a part of an income-producing entity and it is difficult to estimate the future income stream of an equipment from the total income. Since the bases for estsimating the market and income evidences of value are not available for many industrial property valuation, the cost evidence is frequently used. This is supported by the Iowa Department of Revenue:

> After considerable study it was determined that the cost approach should be used to determine the fair value of industrial machinery and equipment [5].

As such, data estimating the cost evidence of value are usually available and a reliable evidence of value for industrial property can be obtained. The procedure for developing cost evidences of value is the subject of this study.

II. REVIEW OF PREVIOUS WORK

The valuation procedure proposed in this dissertation begins with the concept of depreciation. First, a definition of depreciation will be discussed.

A fundamental notion of depreciation is the measure of inferiority of subject property compared to a new substitute. Perhaps one of the most useful definitions is that adopted by the Depreciation Committee of the National Association of Railroad and Utilities Commissioners [6]:

> Depreciation is the expiration or consumption, in whole or in part, of service life, capacity or utility of property resulting from the action of one or more of the forces operating to bring about the retirement of such property from service; the forces so operating include wear and tear, decay, action of the elements, inadequacy, obsolescence, and public requirements; depreciation results in a cost of service.

Also, R. Winfrey defined depreciation as [7]:

> Depreciation is the loss in value of a group of property resulting from a decrease in its ability or capacity to perform present and future service.

As a means of estimating depreciation, several methods have been developed for the proper allocation of the total depreciation over the periods of life. Some of these are straight line, sinking fund, present worth, declining balance, and sum-of-digits methods.

Of all these methods of depreciation, the present worth depreciation method is used in this dissertation because it is closely related to the economic theory of value and the slope of depreciation can be variable in this method, while the others represent only one slope of depreciation. A brief summary of the present worth depreciation method and significant previous developments will be presented.

A. Present Worth Depreciation Method

According to Marston et al. [1], the present worth method is defined as follows:

> This present worth actual depreciation principle is that the depreciated value of an industrial property unit, at any date during its service life, is the present worth at that date of the probable future operation returns yet to be earned by its probable future services.

The operation returns referred to in this quote are the after tax cash flows, that is, operating revenues, less cash operating costs, and cash income tax payments. The operation returns of a property were also defined as including both the periodic depreciation and the net return on the depreciated value [7]. Marston et al. [1] state:

> For a property to justify its economic existence, its annual operation return should be at least sufficient to repay its yearly depreciation allocation and, in addition, pay each year a fair return on the allocated base of the unit at the beginning of the year [1].

It is seen that the present worth method is based on the fundamental notion of value that the value of a property is the present worth of its probable future services.

The depreciation in this method is measured by the amount of decrease in the present worth of these operation returns as the unit increases in age. As the unit becomes older in service, the number of expected future returns decreases, and this results in a lessening of present value with an increase in age [8].

B. Mathematical Expression of Present Worth Method

The mathematical derivation of the present worth method is based on discounting the anticipated future annual operation returns and the estimated salvage value at retirement of a property. The discount rate used was specified as the fair rate of return for the property. These derivations are presented by Marston et al. [1] and Winfrey [8]. Winfrey's derivation is more detailed, but the result is the same as that of Marston's. A brief summary will be included for this dissertation.

The derivation of the formula proceeds on the bases of finding the present worth of the probable future operation returns, R:

$$V_n = V_{nd} + V_s = \frac{R_1}{(1+r)^1} + \frac{R_2}{(1+r)^2} + \frac{R_3}{(1+r)^3} + \dots + \frac{R_n}{(1+r)^n} + \frac{V_s}{(1+r)^n} \quad . \tag{2.1}$$

where,

R_i = operation return for age n,

V_n = the unit's present value at age 0,

V_{nd} = the unit's depreciable value new,

V_s = salvage value,

r = the fair rate of net return on entire property, and

n = the unit's probable life in years.

If all operation returns, R, are assumed to be uniform, then this R can be represented as follows.

$$R = V_{nd} \frac{r(1+r)^n}{(1+r)^n - 1} + rV_s \tag{2.2}$$

And the equation for the value new and the equation for the present value at age x, which is the present worth of the remaining annual operation returns plus the present worth of the salvage value, may be written as follows from Equation (2.1).

$$V_n = V_{nd} + V_s = R \frac{(1+r)^n - 1}{r(1+r)^{n-x}} + \frac{V_s}{(1+r)^n} \tag{2.3}$$

$$V_p = R \frac{(1+r)^{n-x} - 1}{r(1+r)^{n-x}} + \frac{V_s}{(1+r)^{n-x}} \tag{2.4}$$

where,

V_p = the unit's present value at age x, and

x = the unit's service age in year.

The resulting equation for the present value at age x after substituting the value of R from Equation (2.2) in Equation (2.4) is:

$$V_p = V_{nd} \frac{(1+r)^n - (1+r)^x}{(1+r)^n - 1} + V_s , \tag{2.5}$$

The condition percent term, defined by Marston and Agg [9], is 100 times the Ratio of its present depreciable value divided by its depreciable value when new. Marston and Agg continued their definition by noting that four differing applications of the term exist. These are the condition percents for: 1) a single property unit, 2) an average survivor unit of an age-group, 3) all survivors of an age-group, and 4) all units in service from an age-group where continued renewals maintain a constant population [8]. Though the basic definition of the condition percent does not change for these applications, the method of computing the condition percent value does.

Winfrey defined the condition percent in terms similar to Marston and Agg in a later publication [8]. Tables giving the condition percent values for properties at different ages for differing probable lives and discount rates, and based on the assumption of uniform annual operation returns, were also published at a later date [7]. In another later publication coauthored by Marston, Winfrey, and Hempstead [1], the condition percent term was called the "expectancy-life factor."

When derived for a unit of property, the mathematical expression for the condition percent factor is:

$$C = \frac{(1+r)^n - (1+r)^x}{(1+r)^n - 1} \qquad (2.6)$$

where,

C = condition percent factor,

r = annual net rate of return,

n = probable life of unit in years, and

x = age of unit in years.

In common usage, the condition percent factor (C) is often multiplied by 100, thereby expressing it as a percentage; it is referred to simply as the condition percent (C_p).

C. Declining Operation Returns

Though the present-worth principle, as stated earlier, does not depend on the existence of a uniform annual operation return stream, the derivation of Equations (2.2-2.4) is based on the simplifying assumption of uniform annual operation returns. Convincing evidence exists [10], however, that operation returns may decrease with age for many properties rather than remaining constant.

The existence of decreasing operation returns was noted by Terborgh [10] in a book containing analyzed data from several sources having particular relevance to decreasing operation returns. Though the primary thrust of Terborgh's book concerned equipment replacement policy, he included two sets of charts showing a decrease in the quantity of measured service and an increase in repair costs as equipment ages. Figures 2.1 and 2.2 illustrate the two sets of charts. The decreasing quantity of service and increasing repair costs translated directly into a decreasing stream of operation returns, or after-tax-cash-flows, over the life span of an equipment unit. Both the sinking fund method, by definition, and the present worth method, by practice, utilized a constant level of operation returns to compute the amount of accrued depreciation. Therefore, Terborgh's studies suggest that the use of depreciation methods which are based on the assumption of uniform operation returns would not be appropriate.

Further support for the existence of a decreasing operation returns stream was given by Marston et al. As presented by this source, the

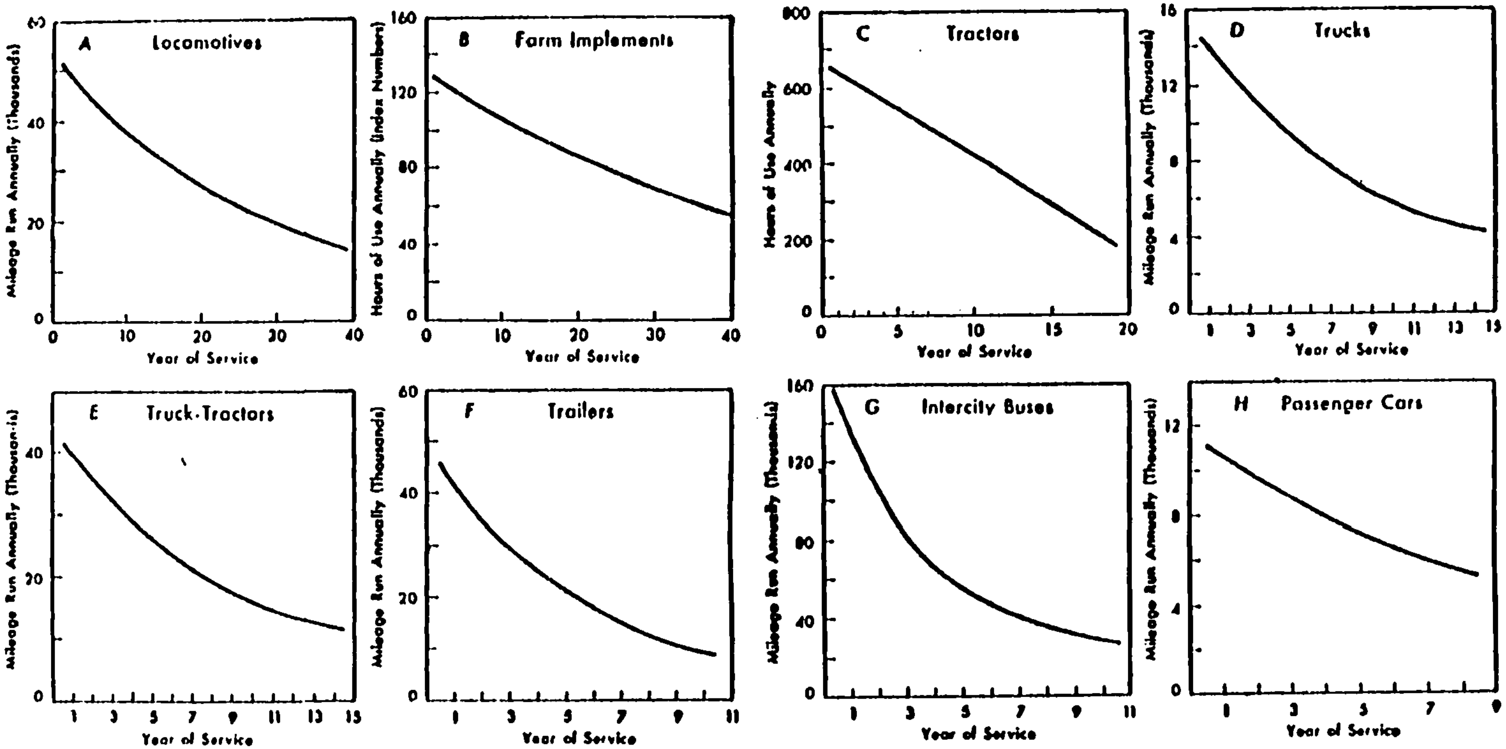

Figure 2.1. Relation between age and intensity of use of eight classes of equipment [10]

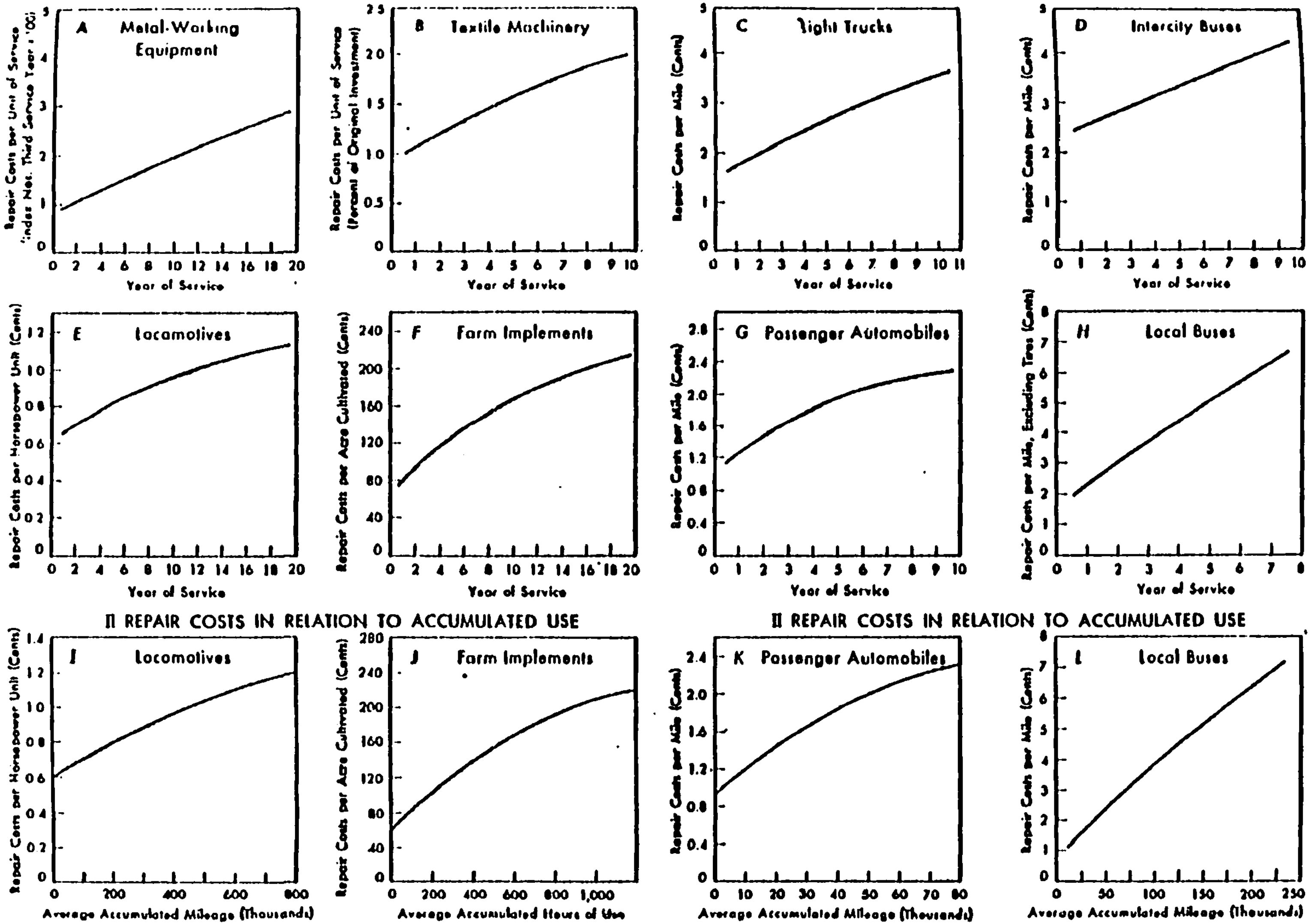

Figure 2.2. Relation between age and repair cost of service for twelve classes of equipment [10]

decrease was "... caused by lowered efficiency of the property, lowered output capacity, increased running costs, intermittent (stand-by) service, and operation at less than normal capacity" [1]. Recently, Griffith [11] also states the existence of decreasing operation returns in his economic depreciation model that are caused by the decreasing revenue and increasing operating costs with the passage of time. The existence of decreasing, as opposed to constant, operation returns has, therefore, been recognized by a number of authoritative sources for many years.

D. Methods of Handling Declining Operation Returns

The existence of decreasing operation returns has resulted in the need for some means of adjusting the formula used to find computed present value. Past methods for handling this nonuniformity have varied. Marston and Agg introduced a "probable future operations return ratio" (PFORR) into their equation for present value. The resultant expression then became:

$$V_p = (V_{nd})\left(\frac{\text{condition percent}}{100}\right)(\text{PFORR}) + V_s \qquad (2.7)$$

In a later source, Marston et al. defined a similar term, but called it a "service factor." In discussing the service factor, this source stated:

> The service factor may be less than unity or greater than unity; its main function is to compensate, when necessary, for failure of the expectancy factor to produce the desired adjustment of the base new to current conditions [1].

The service factor was then inserted into Equation (2.7) in the same manner and with the same resulting expression as the PFORR terms.

Due to the very subjective method of selecting the service factor or PFORR, most appraisers and valuation engineers have simply ignored the term (or equivalently, set the value to unity) in the equation. The result has been to carry the initial assumption of a uniform annual operation return stream through to final cost evidence determination.

An alternate method for handling the nonuniform operation returns was proposed by Elfar [12]. This method incorporated a progression rate term, called a "T-factor," into the derivation of the condition percent factor. The T-factor was related to the operation return stream as shown in the following expression:

$$R_x = R_1 \frac{T^N - T^{x-1}}{T^N - 1} \tag{2.8}$$

where

N = probable life of property unit or frequency group in half-year intervals,

x = age of unit or property group in half-year intervals,

R_x = operation return for age interval quantity, and

T = progression rate of operation returns.

The purpose of the T-factor was to incorporate the effects of a variety of operation return streams, including the uniform case, into the present worth determination of value. The result of the derivation was a "modified condition percent factor" that was used in a manner similar to the original condition percent factor except that the modified condition percent factor incorporated the original condition percent factor with the service factor or PFORR term.

Since the reconsideration of the Elfar model and its modified condition percent factor is one of the important subjects of this dissertation, a summary and brief discussion of the equations derived by Elfar are presented. A detailed derivation of the valuation model can be found in either Elfar's thesis [12] or Cowles-Elfar's paper [13].

Beginning with the present-worth principle as stated by Marston, and incorporating Equation (2.8) at the appropriate point, Elfar derived the following expression for the value at any age:

$$V_x = V_N \; [C'_x(1-S)+S[C'_x(1-(p/f)^i_N) + (p/f)^i_{N-x}]] \qquad (2.9)$$

where

V_x = value at age x of a unit or of survivors of a property group,

V_N = value new of a unit or of survivors of a property group,

C'_x = modified condition percent factor of age x,

S = salvage ratio = (V_S/V_N),

V_S = estimated net salvage value,

$(p/f)^i_n$ = present worth of a future sum,

i = effective semi-annual discount rate, and

N = probable life of a property unit or a frequency group in half-year intervals.

Although the "modified condition percent factor" is analogous to the condition percent factor derived by Marston, Winfrey, and others, the significant difference is that the condition percent factor is a special case of the more general modified condition percent factor. As derived by Elfar, the general, closed form, mathematical expression for the modified condition percent was:

$$C'_x = \frac{q^{N-x-1}(T+iT^{x-N}) - q^{-1}(T+i) - q^{N-x} + 1}{q^{N-x-1}(T+iT^{-N}) - q^{-x-1}(T+i) - q^{N-x} + q^{-x}} \qquad (2.10)$$

where $q = (1+i)$ and all other terms are as previously defined.

By definition, the progression rate, T, was in the range of $0 < T < \infty$. Likewise, the semi-annual rate of return, i, was some value in the range of $0 < i < \infty$. Within these two ranges, however, the two terms may assume values that resulted in unique forms of the general expressions. These special cases, as defined by Elfar [12], occurred when:

$T = 1$ and $i > 0$,
$T = \infty$ and $i > 0$,
$T = 1$ and $i = 0$,
$T = \infty$ and $i = 0$, and
$T < \infty$ and $i = 0$.

By selecting the correct equation for special cases, the modified condition percent factor and corresponding value at any age could be calculated based on the present-worth principle.

E. Procedure to Estimate the T-factor Value

The testing of a general procedure, proposed by Whelan [14], to estimate T-factor values depends on establishing a relationship between the rate at which operation returns decrease and the rate at which depreciation accrues. The rate of decrease in operation returns is reflected in the estimated T-factor value, whereas the amount of depreciation accrued reflects the decrease of value at any age.

Two estimation approaches were proposed by Whelan [14]. The first of these, referred to as the "Ratio" method, uses a ratio of the observed operation return from a property for the first year, R_1, and the observed return for the age interval x-1 to x, R_x, in the estimation of an appropriate T. The second approach, referred to as the "Delta" method, relates the difference between R_1 and R_x with the determination of T. The explanation and derivation of these two methods for a subject property are presented below.

The Ratio method is based on the premise that a decreasing Ratio of operation returns was related to an increasing depreciation accrual rate. The operation return Ratio was specified as being R_x divided by R_1. The Ratio method was tested with the information obtained from a major oil refining company [15]. The returns measured were operation returns for an existing and a new refinery given in units of net dollars per year. A Ratio of the existing refinery's net return (R_x) to a new refinery's returns (R_1) was then formed. The resulting Ratio was set equal to a function of the progression rate in accordance with the Ratio method equation. After the amount of the net returns and the probable life were estimated, the estimated T-factor was calculated for a specified age.

Derivation of the Ratio method equation begins with the basic definition. Then, a slightly rearranged form of Equation (2.8) is substituted into the expression. The result is as follows:

$$\text{RATIO} = R_x/R_1 = \frac{T^N - T^{x-1}}{T^N - 1} . \tag{2.11}$$

If values for R_1, R_x, N, and x are either known or can be estimated, a value for the T-factor can be computed by trial and error.

Though the Ratio method offers an ideal approach and produces estimates of T-factor values with a reasonable degree of judgment, it is limited to the valuation of income producing entities where it is possible to estimate of annual operation returns. Adequate information to apply the Ratio method to industrial equipment is normally not available because most subject properties are components of an income producing entity rather than a whole. In these cases, the returns relate to the whole and normally cannot be assigned to the components outside of arbitrary allocation. The more generally applicable Delta method does offer an alternative for estimating T-factor values for these component properties.

1. Delta method

The assumption for the Delta method is that each piece of industrial equipment produces a constant level of annual gross revenue, G in Figure 2.3, from the date of installation onward. Annual costs of generating this revenue need to be deducted to give the true annual operation return. These costs would include actual cash outlays plus penalties in the form of higher costs resulting from such things as obsolescence, less usage, or property inadequacies. Alternately, the penalties can be thought of in terms of diminished revenues or increased costs.

For individual property items or groups of items not all of the costs may be known or even estimable. In this case, the sum of the unknown costs, designated as B, is assumed to be constant over the property's life. Measurable operating costs are, in turn, denoted by P_x. The operation return R_x, for any year, x, is by definition:

$$R_x = G - B - P_x \quad . \tag{2.12}$$

In general, it is expected that R_x will diminish with age and it is assumed that a significant portion of that diminution can be sensed from observing the increasing measurable annual costs, P_x, for progressively older ages. Schematically, this is shown in Figure 2.3.

Application of the Delta method is based on the assumption that, though net operation returns are not known, periodic values of P_x can be estimated. This assumption forms the basis of the following development of the Delta method.

The value of annual gross revenue at the end of the first year is

$$G = B + P_1 + R_1 \tag{2.13}$$

Since G is assumed to be constant,

$$B + P_1 + R_1 = B + P_x + R_x \; . \tag{2.14}$$

Rearranging this expression, and eliminating the unknown term B, results

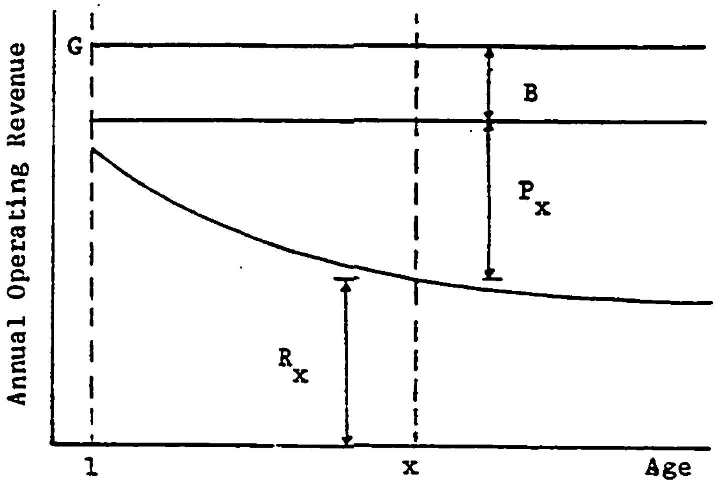

Figure 2.3. Schematic diagram of Delta method concept

in the following expressions:

$$P_x - P_1 = R_1 - R_x \, . \tag{2.15}$$

If the quantity $(P_x - P_1)$ is defined as the term DELTA, and Equation (2.8) is substituted into Equation (2.15), then,

$$\text{DELTA} = R_1 - R \, \frac{T^N - T^{x-1}}{T^N - 1} \, . \tag{2.16}$$

If the derivation procedures presented by Elfar, Whelan, and Cowles and Whelan are used, a closed form equation results:

$$\frac{\text{DELTA}}{V_{new}} = \frac{(q^N - S)(T^{x-1} - 1)(T - q)(i)}{T^N(Tq^N - T - q^{N+1} + 1) + (iq^N)} \, . \tag{2.17}$$

Equation (2.17) is the general expression used to estimate the value of

T for a given property. The DELTA/V_{new} term will be referred to as the Delta Ratio. As in the derivation of the original model by Elfar, a number of special cases occur when $T = 1$, $T = \infty$, and/or $i = 0$.

2. Application of the Delta method

The Delta method proposed by Whelan [14] involves several steps, the first being to quantify P_x, the measurable annual reduction of gross revenues. Significant components of P_x include repair and maintenance costs, downtime expenses, productivity losses, and obsolescence penalties. Of these, repair and maintenance costs are normally the most readily available from a company's cost accounting records. Downtime expenses and productivity losses may or may not be available from the cost accounting records. If not, then these amounts must be estimated from whatever data might be available. The cost of obsolescence is rarely recorded as a part of a cost accounting system. It must, therefore, also be estimated on the basis of judgment or inferred indirectly from costs experienced with similar new equipment. When the major components of P_x have been measured, computed, or estimated for as many age intervals or years as possible, they are adjusted to constant dollar units and summed by year of occurrence.

The second step involves the determination of the DELTA terms, the Delta Ratio (DELTA/V_{new}), and the probable service life. The Delta term for a year or age interval x is simply the difference between P_x and P_1. The value new term is evidenced by the original cost, trended or untrended as appropriate, or the current replacement or reproduction cost of the property. With values of DELTA and V_{new} in hand, the analyst can then compute the Delta Ratio for each age interval. The property's probable service life can be estimated using Iowa type survivor curves. The resulting T-factor values from the DELTA method for the property were then compared with the T-factor values of the market evidence data.

III. OBJECTIVES

The determination of value of an industrial property is important for several reasons. Unfortunately, in the majority of the situations, market evidences, which are the most reliable evidences of value, do not exist. Therefore, the objective of this dissertation is to develop a procedure to aid in estimating the value of such industrial property and equipment with the passage of age. For this study, the present worth depreciation method which relates with the economic theory of value was used within the cost approach. In the present worth depreciation method, the operation returns are assumed to be uniform for simplicity, but most of the actual operation returns of the industrial properties appear to be declining with age. Thus, the basic premise for this study is that there usually is a declining operation return over the ages of most property items or operating units. As the reasons for declining operation returns, increased repair and maintenance costs, decreased usage and the penalties of obsolescence are studied in this dissertation.

Elfar proposed a model in which the variable declining operation returns could be generated through the use of the T-factor value. The present worth method combined with the Elfar model produced the new equations of condition percent and the present value of a property. And Whelan proved the validity of the Elfar model with the Delta method he proposed.

However, the Elfar model is complicated if properly applied and seems to have difficulty in matching the value curve with the market value over all ages. And further study is desired to demonstrate the validity of the declining operation return model. In addition, the Delta method seems to have difficulty in measuring the actual loss in property value.

Thus, the objectives of this study are as follows:

1. Refine the Elfar model or develop a simple modified equation or presentation which is realistic and easy to apply.
2. Continue the proof of the Elfar model with a variety of real world data.
3. Modify the Delta method so that it might produce results which better match market evidence data.
4. Develop practical microcomputer program and standard tables for convenient usage.

IV. MODEL DEVELOPMENT

The variation of the annual service worth of industrial property with the passage of time is important for evidence of values at different ages, especially at ages other than zero, since the fundamental basis of value of a property is the present worth of its probable future services. At age zero, the value of the equipment is assumed to be evidenced by its cost new. However, for the ages other than zero, it is desired to establish evidences of value of industrial property, especially when annual returns cannot be segregated from those of the whole enterprise. Here, the properly calculated annual operation returns, composed of annual recovery of the investment plus the annual return on the unrecovered investment, are considered as the measures of the annual service worths of the machinery and equipment in question. As discussed earlier, the operation returns of a property unit through its service life could be uniform or variable from period to period. It was also stated by Marston et al. [1], however, the calculation of an equivalent operation return from an actual variable operation return pattern produced incorrect results for the condition percent and the value of property at any age other than zero. Thus, the annual operation returns of the machinery and equipment are assumed to decline with the passage of time. The reasons for this decline could be illustrated by a machine which produces the same quantity of the product while its annual operating costs are rising, or a machine that produces increasing percentage of rejects as it ages, or a machine which is superseded on the market by better and more economical substitutes which relatively increase the former's cost of operation.

Accordingly, it was natural to try to suggest a valuation model which offers the possibility of representing the case of declining operation returns with the age of property.

In this context, the several models for the declining operation returns were collected and proposed.

A. Model Proposed

The conditions of selecting the best model among these models are divided into three categories. The first one is the model should represent the proper declining of the operation returns of industrial properties in general. The operation returns of different industrial properties are different from each other. Thus, the model should be able to include and represent the general change of operation returns with ages of a property. The second one is that the values of each age which are derived from the operation returns model have to fit well to the existing real market values of properties. This derivation is done by the present worth principle. The third one is that the model has to be simple and easy to use and apply since this model has to be made for practical usage in industry.

The following eight models were developed and examined with trial and error method.

1. $$\frac{R_x}{R_1} = (1+T)^{x-1} \qquad (4.1)$$

This model is used in engineering economy problems involving geometric progression [16].

2. $$\frac{R_x}{R_1} = \frac{N-T(x-1)}{N} \tag{4.2}$$

This model is based on a straight line decline.

3. $$\frac{R_x}{R_1} = \left(1 - \frac{x-1}{N}\right)^{\frac{1}{T}} \tag{4.3}$$

4. $$\frac{R_x}{R_1} = \frac{T^N - T^{x-1}}{T^N - 1} \tag{4.4}$$

This model is used in Elfar's thesis to match market value with the present worth method.

5. $$\frac{R_x}{R_1} = 1 - \left(\frac{x-1}{N}\right)^T \tag{4.5}$$

6. $$\frac{R_x}{R_1} = \frac{T^N - T^{x-1}}{T^N - 1} - .1 \sin \frac{6(x-1)}{N} \tag{4.6}$$

7. $$\frac{R_x}{R_1} = 1 - \left(\frac{x-1}{N}\right)^T - .1 \sin \frac{5(x-1)}{N} \tag{4.7}$$

8. $$\frac{R_x}{R_1} = 1 - \left(\frac{.7(x-1)}{N}\right)^K \quad ; \quad \frac{R_x}{R_1} = 0 \text{ at } x = N+1 \tag{4.8}$$

The first, second, and third models, after the investigation and the comparison with the actual real world data, are satisfactory in the first and third categories but have defects for the second criterion category. The fourth model proposed by Elfar [12] results in better fitting in the second category but that was not enough. Because when the property is relatively new, the value at age x derived by using the Elfar model is bigger than market evidence value and in later ages, the value at age x is smaller than market evidence value all the time. Besides, the value of age x becomes smaller than indicated salvage value of the property in later ages if the average life of a property is somewhat larger than ten years. This will be shown later in Figure 4.10 and is one of the main reasons to develop alternate models to the Elfar proposal. Also, the model is not simple enough to satisfy the third category. Among the last four models, model No. 7 is almost perfectly fitted to the market value but the model induced the sine curve and became complicated. But, model No. 8 is simple and the fitting is better than others; thus, this model No. 8 is acceptable for all three categories. As a result, the model No. 8 was chosen and named "Y model" to distinguish it from the Elfar model. As a slope factor, K will be used instead of T. In the Y model, the constant, 0.7, was determined by the investigation and the comparison with the actual data collected for this study with the trial and error method. Various numbers were tested to find the best matching constant. It was found that the larger the constant was, the greater the drop of the operation returns in the first year. In this context, 0.7 was found as the best constant for the model of this study purpose.

B. Y Model Application

With the Y model, it will be shown that, in infinite mode, (when K =) it represents the case of uniform periodic operation returns, and in normal mode, it represents the rapid drop of value in the very earlier ages and keeps slowly declining until the last age. In this model $R_{N+1} = 0$ and the last period that the property would have an operation return is N. The slope factor K can take any value greater than zero.

According to the present worth principle, the value of a property is the present worth of its future operation returns. The value new, V_N, can then be written as follows:

$$V_N = R_1(p/f)^{i}_{1} + R_2(p/f)^{i}_{2} + \dots + R_N(p/f)^{i}_{N} + V_S(p/f)^{i}_{N} \qquad (4.9)$$

$$= R_1(p/f)^{i}_{1} + R_1\ [1-(\frac{.7}{N})^K](p/f)^{i}_{2}\ + \dots$$

$$+ R_1\ [1-(\frac{.7(N-1)}{N})^K](p/f)^{i}_{N}\ + V_S(p/f)^{i}_{N} \qquad (4.10)$$

$$= R_1 \sum_{M=1}^{N}\ [1-(\frac{.7(M-1)}{N})^K](p/f)^{i}_{M}\ + V_S(p/f)^{i}_{N} \qquad (4.11)$$

from which R_1 can be expressed as:

$$R_1 = \frac{V_N - V_S(p/f)^{i}_{N}}{\sum\limits_{M=1}^{N} [1-(\frac{.7(M-1)}{N})^K]\ (p/f)^{i}_{M}} \qquad (4.12)$$

Now, applying the present worth principle at any other age x, V_x will be:

$$V_x = R_{x+1}(p/f)^{i}_{1} + R_{x+2}(p/f)^{i}_{2} + \dots + R_N(p/f)^{i}_{N-x}$$

$$+ V_S(p/f)^{i}_{N-x} \quad . \qquad (4.13)$$

Substituting the values of R_{x+1}, R_{x+2}, etc., using the model, V_x becomes:

$$V_x = R_1\ [1 - (\frac{.7(x)}{N})^K]\ (p/f)^{i}_{1} + R_1\ [1-(\frac{.7(x+1)}{N})^K]\ (p/f)^{i}_{2}$$

$$+ R_1\ [1-(\frac{.7(N-1)}{N})^K](p/f)^{i}_{N-x} + V_S(p/f)^{i}_{N-x} \qquad (4.14)$$

$$= R_1 \sum_{L=1}^{N-x}[(1-(\frac{.7(L+x-1)}{N})^K](p/f)^{i}_{L} \quad + V_S(p/f)^{i}_{N-x} \qquad (4.15)$$

Substituting the value of R_1, the expression for V_x will be:

$$V_x = V_N - V_S(p/f)^{i}_{N}\ \frac{\sum_{L=1}^{N-x} [1-(\frac{.7(L+x-1)}{N})^K](p/f)^{i}_{L}}{\sum_{M=1}^{N} [1-(\frac{.7(M-1)}{N})^K](p/f)^{i}_{M}} + V_S(p/f)^{i}_{N-x} \qquad (4.16)$$

$$= V_N - V_S(p/f)^{i}_{N}C'_x + V_S(p/f)^{i}_{N-x} \qquad (4.17)$$

C'_x will be called the "modified condition percent factor of the Y model" to differentiate it from the condition percent factor derived by Winfrey

[8] for the specific case of uniform annual operation returns. C'_x for the proposed model is expressed as:

$$C'_x = \frac{\sum_{L=1}^{N-x} \left[1-\left(\frac{.7(L+x-1)}{N}\right)^K\right](p/f)^{i}_{L}}{\sum_{M=1}^{N} \left[1-\left(\frac{.7(M-1)}{N}\right)^K\right](p/f)^{i}_{M}} \quad . \qquad (4.18)$$

The above model is a general expression and the proposed model could represent a multitude of situations. Slope factor K values range from just above zero to infinity. At the same time, discount rates of zero or larger can be applied. Also, salvage ratios can be positive, negative, or zero.

B. Y Model Characteristics

It is important to find the effect of a change of parameters and to understand the proposed model. To understand the model characteristics helps to improve the application and reliability of the model. The variables are K value, discount rate, average life, and salvage rate. In this section, the changes of the ratio of operation return (R_x/R_1), the condition percent factor (C'_x) and the value of property (V_x) through its life will be shown and discussed.

1. Ratio of operation return

The ratio of operation return shows the relationship between the first operation return and the operation return of age x. This ratio is as follows in the proposed model,

$$R_x/R_1 = 1 - (\frac{.7(x-1)}{N})^K \quad .$$

In this model, the parameters are K and N. The variations of the ratio according to the change of K and N are presented in Figures 4.1 and 4.2. The actual values of K are limited to non-negative values. In the case of zero, the graph becomes the horizontal axis and in the case of infinity, the graph will be always the horizontal line of 1. In this graph, a condition is that the ratio should be 0 at the end of life. The interesting observation of these variations of K value is that the lower the K value, the steeper the graph of the early ages. Thus, usually the ratio graph is steep in the early ages and in the other ages the graph declines slower compared to the Elfar model as shown in Figure 4.10. That is the reason why the proposed model fits better to the market evidence than the Elfar model.

In this proposed model, it is interesting to note that there are large ratio changes at the first and the last age. Also, the intervals between the graphs become relatively smaller as the K values become larger. For example, the interval between the graphs of .2 of K and .4 of K is bigger than the interval of .8 and 1.0, even though the differences of figures are same. It is seen that the ratio of operation return increases with the increase of K value.

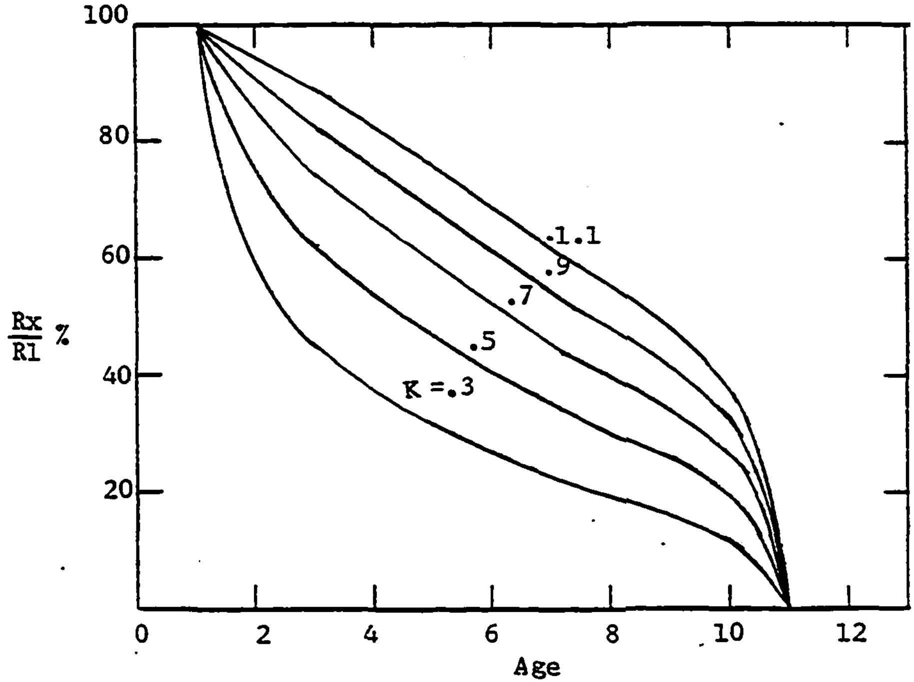

Figure 4.1. Y model graph with variation of K value

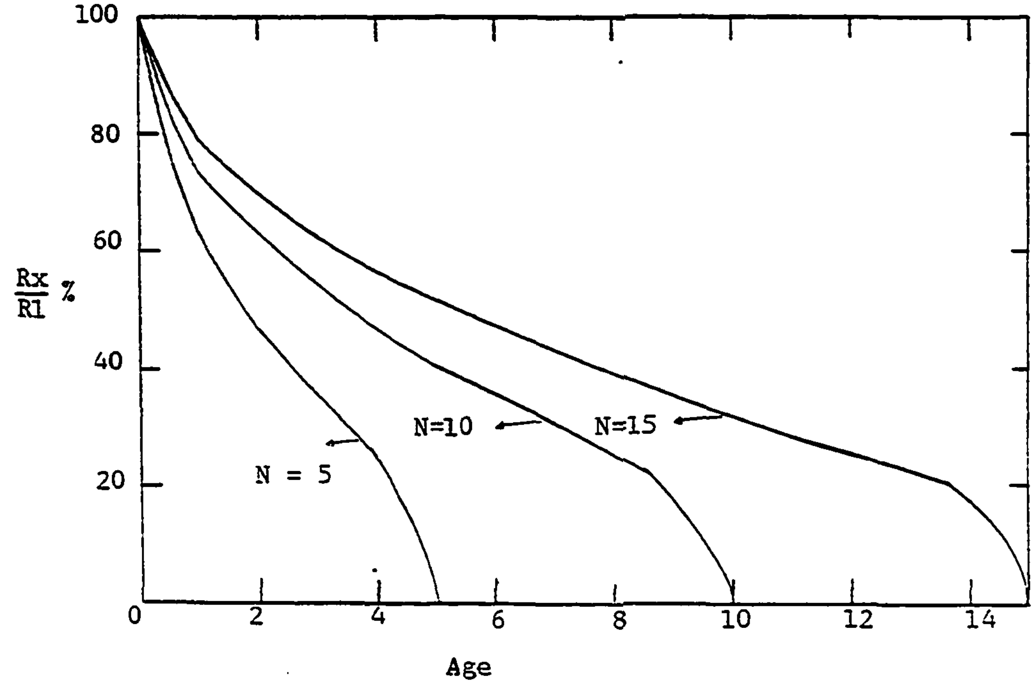

Figure 4.2. Y model graph with variation of N

2. Modified condition percent

The equation of modified condition percent represented in Equation (4.18) is:

$$C_x' = \frac{\sum_{L=1}^{N-x} [1-(\frac{.7(L+x-1)}{N})^K](p/f)_L^i}{\sum_{M=1}^{N} [1-(\frac{.7(M-1)}{N})^K](p/f)_M^i}$$

In this modified condition percent, the parameters are K, N, and I. Thus, it is necessary to make graphs with the change of these parameters. Figures 4.3, 4.4, and 4.5 show the change of modified condition percent values according to the change of K, N, and I, respectively. A change in K results in considerable change in the C_x' graph, but changes in N and I have little effects. That means the effect of discount rate almost zero within the common range of 0 to 0.1 and the effect of length of life on C_x' is quite small since the length of the industrial property is usually in between 10 and 20.

3. Value of property using Y model

The equation of V_x, value of property at age x, as given by the proposed model is:

$$V_x = V_N - V_S(p/f)_N^i \frac{\sum_{L=1}^{N-x} [1-(\frac{.7(L+x-1)}{N})^K](p/f)_L^i}{\sum_{M=1}^{N} [1-(\frac{.7(M-1)}{N})^K](p/f)_M^i} + V_S(p/f)_{N-x}^i$$

The parameters which can have an effect on the value of age x are K, N, S, and I. Figures 4.6, 4.7, and 4.8 show the effect of changing the K value, salvage ratio, and discount rate, respectively. In Figure 4.6, N is assumed to be 10 years, S is 10 percent, and I is 7 percent. The effect of changing the K value is quite similar to the case of modified condition percent. That is, the V_x becomes larger as the K value increases. The V_x decreases rapidly in early ages and decreases more slowly in later ages.

Figure 4.7 shows the effect of salvage ratios. It is interesting that the difference between V_x when S=.1 and V_x when S=.2 is very small in early life but becomes greater toward the end of life. This can be

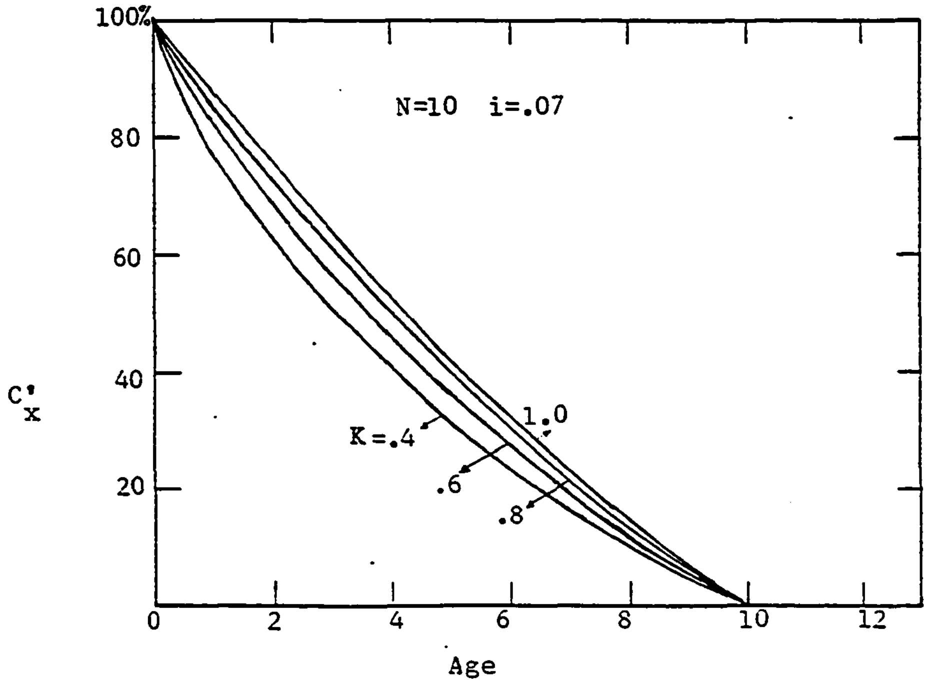

Figure 4.3. Condition percent graph with variation of K value

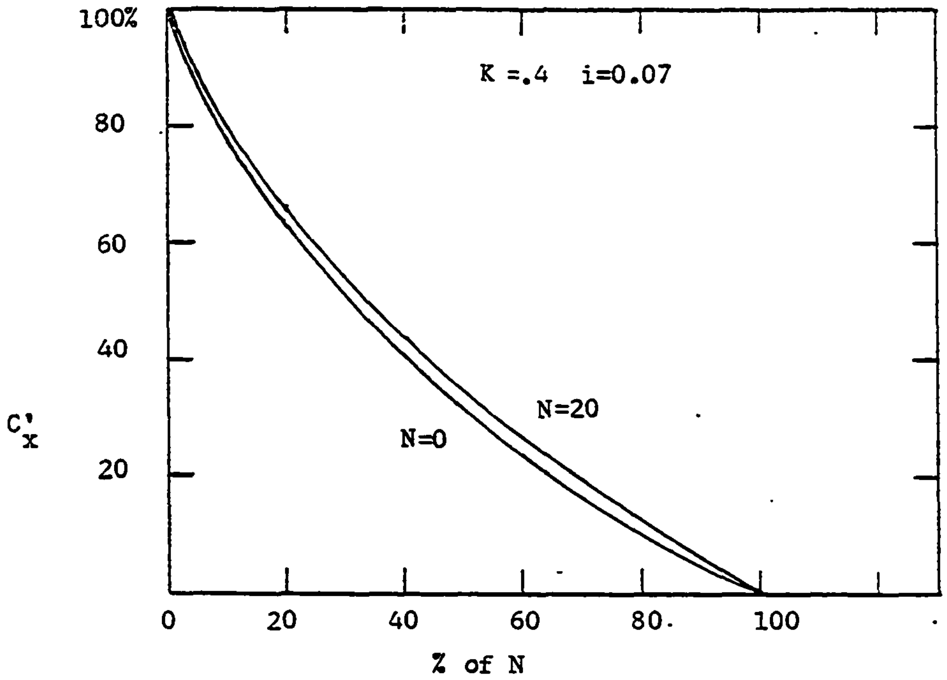

Figure 4.4. Condition percent graph with variation of N

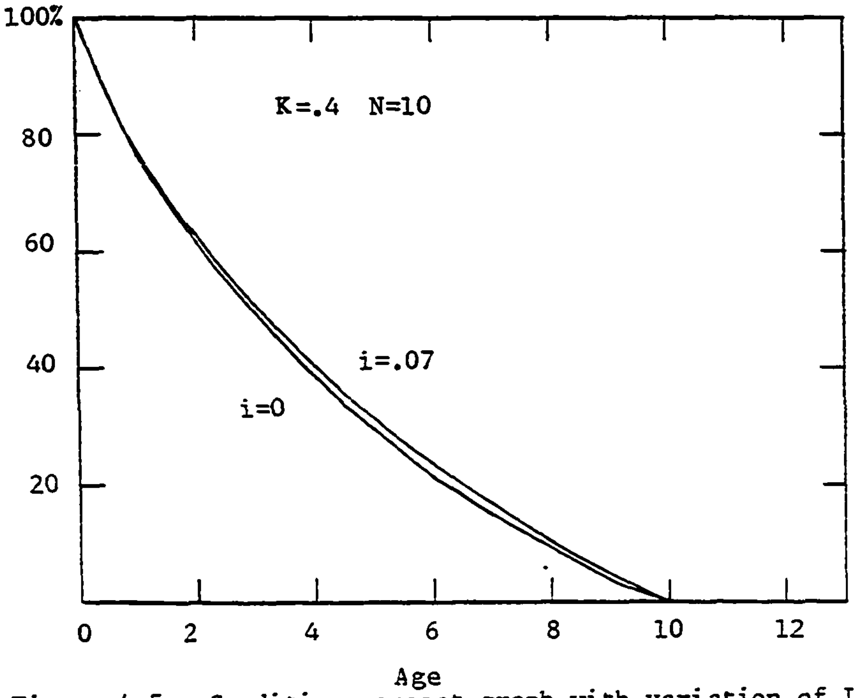

Figure 4.5. Condition percent graph with variation of I

thought of in such a way that if the difference of salvage ratios is small, the difference between V_x values is small, especially in the early life. The effect of discount rate is seen in Figure 4.8. That plot shows the effect of change in discount rate is almost zero. Thus, the effect of the K value on the value at age x is greater than that of salvage ratio or discount rate.

The graphs for the effect of length of life with respect to the percent of age are illustrated in Figure 4.9. Finally, in Figure 4.10, there is the comparison between the V_x graphs which were obtained by using the Elfar model and V_x graph out of the Y model proposed in this dissertation. From this comparison, it is seen that the V_x of Elfar model is greater in the early ages of life and is smaller in the later ages. For example, the V_x graph of the Elfar model at T=.8 is possibly the closest to the V_x graph of the Y model at K=.15. But, because of the difference of the basic model, there is a large gap in the early ages and also in the later ages.

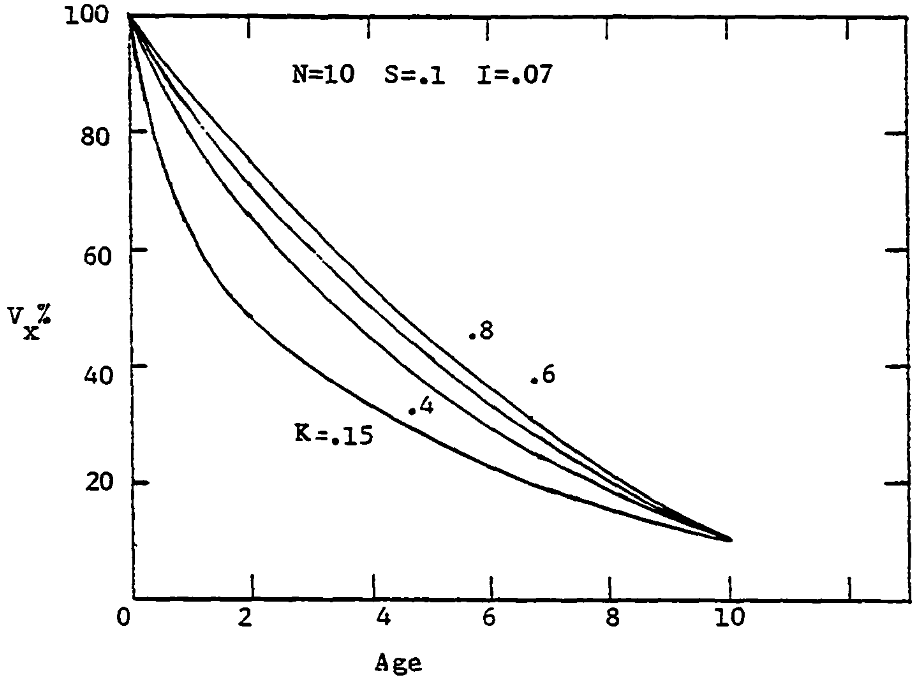

Figure 4.6. Value graph with variation of K value

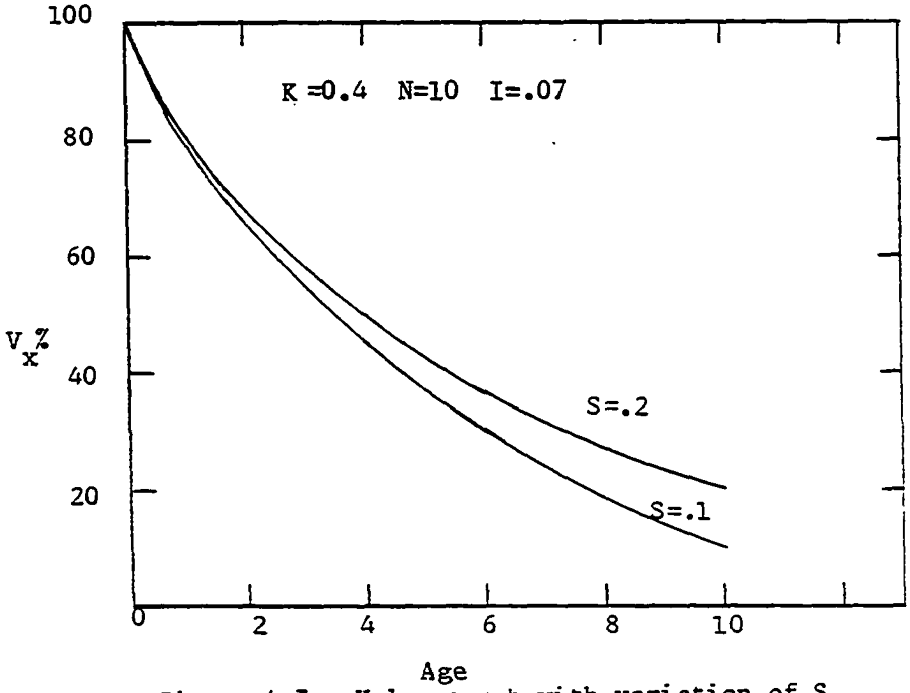

Figure 4.7. Value graph with variation of S

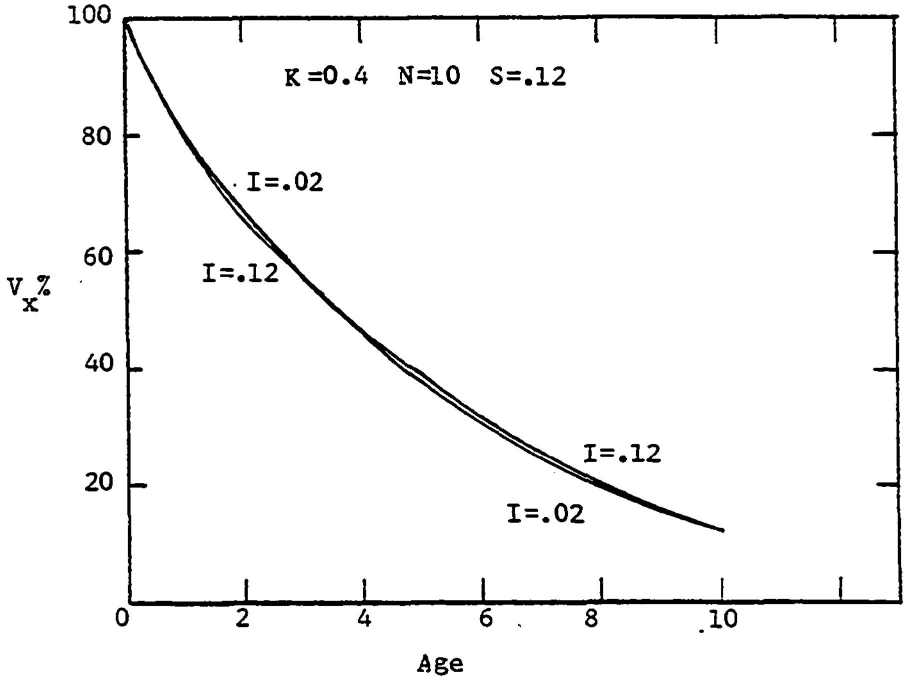

Figure 4.8. Value graph with variation of I

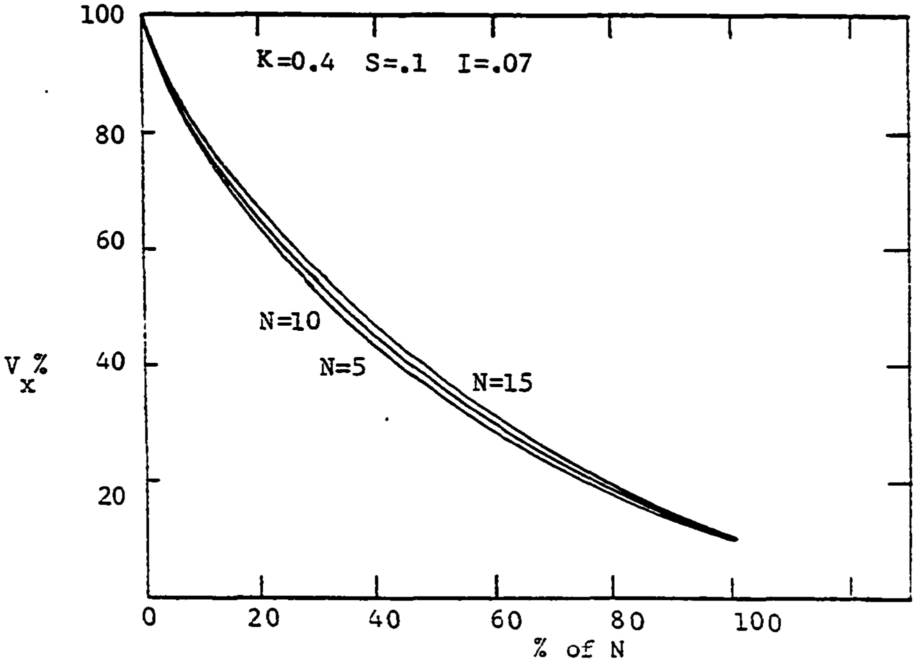

Figure 4.9. Value graph with variation of N

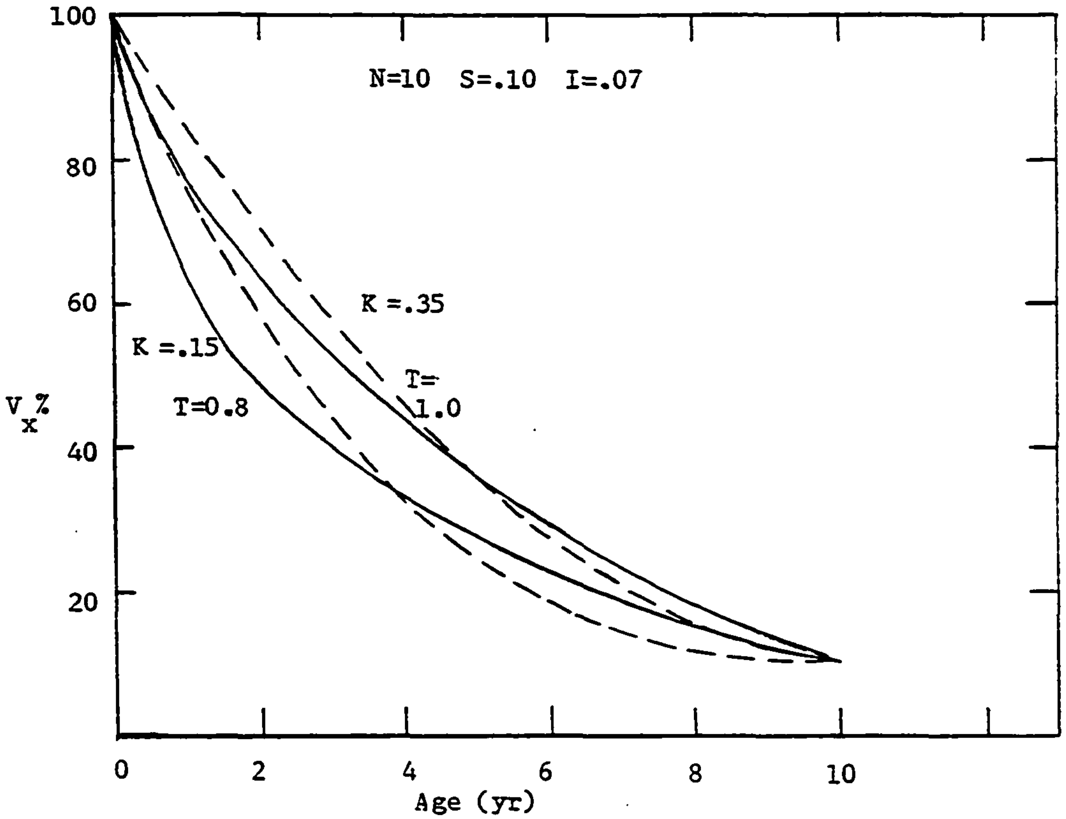

Figure 4.10. Comparison of V_x from Y model and Elfar model

V. PROCEDURE DEVELOPMENT

As has been previously noted by Marston et al. [1], value depreciation results from such causes as increasing repair and maintenance expenditures, decreasing production rates, reduced availability, and accumulating obsolescence. However, it is difficult to measure and record this factor. Thus, a basic assumption is made that the above factors can be condensed into the two factors: the increasing repair and maintenance costs and the decreasing of the service intensity. Griffith [11] also supported this assumption in his study. However, the level of service intensity may not change enough to sufficiently measure the effect of obsolescence.

In contrast to this assumption, Whelan [14], considered mainly the repair and maintenance costs and downtime costs. However, the downtime costs are not always available and are hard to express in actual costs.

A. Estimate of Service Intensity

As described before, the decreasing service intensity is one of the main reasons of the decreasing value with the increase in age. The effect of decreasing service intensity may be larger than the effect of increasing costs of repair and maintenance. Service intensity has the advantage of directly being related to the operation returns with the proportion to the usage hours or mileage operated with ages.

Level of service intensity is influenced by the decreasing production rate, reduced availability, obsolescence, risk, and downtime cost. Because the machine usage hours would be decreased at least as much as the downtime, the influence of the obsolescence might lead to less usage if more modern equipment is available. Also, use of the machine is likely to be avoided as production rates become low. Thus, the decrease of the service intensity could include all of the above factors. The decreasing service intensity is one of the main factors, as well as increasing repair and maintenance costs, for the estimation of the value of a property with respect to its age.

Service intensity is represented as the ratio of yearly usage through ages of property. The biggest yearly ratio usage during ages of property life will be usually 1 and the other yearly usage will be the proportion of biggest one, namely, between 1 and 0. Ideally, the first year usage is the greatest one, if not, the first year usage has to be properly estimated with the smoothed curve of whole life yearly usages.

Service intensity with the respect to the age of the machinery is sometimes hard to obtain. Thus, if the data for specific equipment were not available, the representative intensity graph will be used. For example, each of three kinds of intensity graphs represents the overall intensity for light trucks, middle weight trucks, and heavy trucks, respectively.

B. Composition of Repair and Maintenance Costs with Service Intensity

Since the repair and maintenance costs are measured in dollars, and the usage intensity is the ratio of the yearly usage, the combination of these two is complicated. However, it is necessary to calculate the whole decrease of the operation returns of overall ages.

The assumption of the present worth principle using the symbols is represented as follows:

$$V_N = R_1(p/f)_1^i + R_2(p/f)_2^i + \ldots + R_N(p/f)_N^i + V_S(p/f)_N^i \qquad (5.1)$$

Here, R_x is supposed to be decreased with the age x according to the composition of the service intensity and the repair and maintenance costs. Suppose the intensity graph is f(x) and the repair and maintenance costs divided by value new, is t(x). Then,

$$R_x = G\cdot f(x) - V_N\cdot t(x) \qquad (5.2)$$

where R_x = net operation returns of age x,

G = original returns without reduction of repair and maintenance costs and loss of usage intensity, and

V_N = value when a property was new.

A schematic diagram showing this relationship is shown in Figure 5.1. Substituting the R_x from Equation (5.2) into Equation (5.1), the expression for V_N will be:

$$V_N = \sum_{x=1}^{N} [G\cdot f(x) - V_N\cdot t(x)](p/f)_x^i + V_S(p/f)_N^i \qquad (5.3)$$

Therefore, if all the data, such as V_N, N f(x), t(x), i, and V_s are provided, the amount of G can be obtained. Then, the P_x, the sum of repair and maintenance cost and usage intensity, can be calculated as follows:

$$P_x = G - R_x = G\cdot[1-f(x)] + V_N\cdot t(x) \qquad (5.4)$$

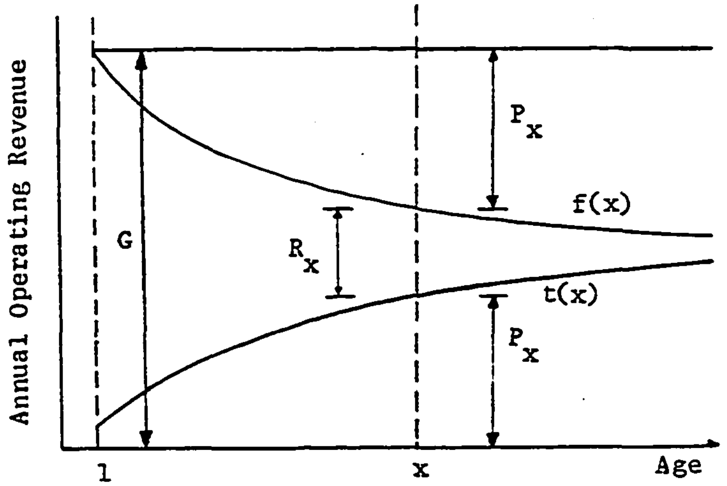

Figure 5.1. Schematic diagram of composition

One can also calculate the value of a property at age x directly from R_x, but the f(x) and t(x) data for the calculation of R_x are not sufficient for the full data until the life, the value at age x from R_x might be acceptable but not reliable. Thus, the reliable data for this calculation is the P_x which are supported by exact data. These P_x data can be applied to the Delta method to obtain a T value.

Derivation of the G from the above Equation (5.3) is as follows:

$$V_N = \sum_{x=1}^{N} [G \cdot f(x) - V_N \cdot t(x)](p/f)_x^i + V_S(p/f)_N^i$$

$$= \sum_{x=1}^{N} [G \cdot f(x) \cdot (p/f)_x^i - V_N \cdot t(x)(p/f)_x^i] + V_S(p/f)_N^i \qquad (5.5)$$

$$= G \sum_{x=1}^{N} f(x)(p/f)_{x}^{i} - V_N \sum_{x=1}^{N} t(x)(p/f)_{x}^{i} + V_S(p/f)_{N}^{i} \qquad (5.6)$$

$$\therefore G \sum_{x=1}^{N} f(x)(p/f)_{x}^{i} = V_N + V_N \sum_{x=1}^{N} t(x)(p/f)_{N}^{i} - V_s(p/f)_{N}^{i} \qquad (5.7)$$

$$\therefore G = \frac{V_N \, [1 + \sum_{x=1}^{N} t(x)(p/f)_{x}^{i} + S(p/f)_{N}^{i}]}{\sum_{x=1}^{N} f(x)(p/f)_{x}^{i}} \qquad (5.8)$$

where S = salvage ratio

$$\therefore \frac{G}{V_N} = \frac{1 + \sum_{x=1}^{N} t(x)(p/f)_{x}^{i} + S(p/f)_{N}^{i}}{\sum_{x=1}^{N} f(x)(p/f)_{x}^{i}} \qquad (5.9)$$

Also,

$$V_x = \sum_{g=x+1}^{N} [G \cdot f(g) - V_N \cdot t(g)](p/f)_{g-x}^{i} + V_S(p/f)_{N-x}^{i} \qquad (5.10)$$

And the P_x is

$$P_x = G \cdot [1 - f(x)] + V_N \cdot t(x)$$

$$= \frac{V_N \, [1 + \sum_{x=1}^{N} t(x)(p/f)_{x}^{i} + S(p/f)_{N}^{i}]}{\sum_{x=1}^{N} f(x)(p/f)^{i}} [1 - f(x)] + V_N \cdot t(x)$$

$$= V_N \left[\frac{[1+\sum_{x=1}^{N} t(x)(p/f)_x^i + S(p/f)_N^i][1-f(x)]}{\sum_{x=1}^{N} f(x)(p/f)^i} + t(x) \right] \tag{5.11}$$

Starting this P_x value, the next procedures are as follows:

1. Finding the K value using standard curves of the Delta method.
2. Applying the K value to the Y model.
3. Finding the value at age x.
4. Comparing the above V_x with the actual market value.

C. Standard Curves for Finding K Values

Standard curves are helpful in finding the K values. They are used as a basis for the fitting of the data which are derived from the P_x of the previous section. The derivation of the standard curve equation was presented by Whelan [14] in detail. Thus, the summary and the differences will be shown.

$$G = R_x + (G-R_x)$$

$$= R_x + P_x \quad .$$

Since G is constant all over the ages,

$$G = R_x + P_x = R_1 + P_1$$

$$\therefore P_x - P_1 = R_1 - R_x$$

Set Delta (Δ) = $P_x - P_1$

Then, $\Delta = R_1 - R_x$. Substituting the Y model to the equation above,

$$= R_1 - R_1 \left[1 - \left(\frac{.7(x-1)}{N}\right)^K\right] \tag{5.12}$$

From the previous chapter, Equation (4.12),

$$R_1 = \frac{V_N - V_S(p/f)^i_N}{\sum_{M=1}^{N}\left[1-\left(\frac{.7(M-1)}{N}\right)^K\right](p/f)^i_M} .$$

Therefore,

$$\Delta = \frac{V_N\ (1-S(p/f)^i_N)}{\sum_{M=1}^{N}\left[1-\left(\frac{.7(M-1)}{N}\right)^K\right](p/f)^i_M} \quad \left[1-\left(1-\left(\frac{.7(x-1)}{N}\right)^K\right)\right]$$

$$= \frac{V_N\ (1-S(p/f)^i_N)}{\sum_{M=1}^{N}\left[1-\left(\frac{.7(M-1)}{N}\right)^K\right](p/f)^i_M} \quad \left(\frac{.7(x-1)}{N}\right)^K$$

$$\therefore \frac{\Delta}{V_N} = \frac{1-S(p/f)^i_N}{\sum_{M=1}^{N}\left[1-\left(\frac{.7(M-1)}{N}\right)^K\right](p/f)^i_M} \quad \left(\frac{.7(x-1)}{N}\right)^K \tag{5.13}$$

where

$$(p/f)^i_N = \left(\frac{1}{1+i}\right)^N$$

From the above Equation (5.13), all the other factors are known except Δ and x. Therefore, Δ/V_N will be in the vertical axis and age x will be in the horizontal axis in the standard curves.

The variation of average life N and salvage value S can provide a variety of standard curves for the Delta method.

D. Procedural Steps for Delta Procedure

The proposed procedure for estimating slope factor values, K, has several steps. The procedure is almost the same for estimating T-factor values. These steps, and a brief explanation of each, are summarized below:

1. Quantify P_x (Periodic Reduction in Operation Returns): The most readily available data concerning the reduction in operation returns are the repair and maintenance records compiled by most companies. Intensity ratios are often not recorded as a part of normal cost accounting procedures, so they may have to be estimated from incomplete records or ready made representative data records. When these major components of P for each period have been computed, measured, or estimated, adjusted to units of constant dollars, and expressed in units of dollars per period, they can be summed by the method of composition of repair and maintenance costs and intensity ratios to determine the total periodic amount of the reduction of operation returns.
2. Compute Δ: The term is the numerical difference between P_1 and P_x, where P_x is the reduction in operation returns for the period x-1 to x. The computation of Δ is accomplished by

subtracting the value of P_1 from successive values of P_X determined in the previous step.

3. Determine V_N: The V_N term is the value of the property when it was new. Normally, the replacement cost is the best indication of the value new term.

4. Compute Δ/V_N: Computation of Delta Ratio is performed for each successive period by dividing the V_N determined in the previous step into the values computed in an earlier step.

5. Determine the Probable Service Life: The probable service life is determined using the Iowa type curves, or other comparable methods, if sufficient life analysis data are available. If not, it must be estimated based on the best information available.

6. Estimate the value of K using a set of Standard Curves: Since the solution of Equation (5.13) of the previous section for a K value is at best a trial and error procedure, a procedure based on the visual matching of observed data to theoretical, calculated results will be used to estimate K values.

VI. PROCEDURE

A number of actual data sets were collected for the test of the proposed model and procedure. The test will be done by the comparisons of K values of market evidences with those obtained by use of the proposed model and procedure, and the validity of them will be estimated by the closeness of K values. For this test, the data should include market value evidences, repair and maintenance cost data, service intensity rate data, and data indicating salvage values, probably average service lives and the value at age 0 for selected industrial properties. In the case of group properties, operating costs, data, and operating revenue data were sought for the substitute of repair and maintenance costs data and service intensity rate data, respectively.

A. Data for Unit Property

The data sets of twenty-two different types of equipment were used for this experiment. These are as follows:

1. Gas tractor
2. Diesel tractor
3. Self-propelled combine
4. Corn picker
5. Forage harvester
6. Hay baler
7. 35-ton truck
8. 50-ton truck
9. D9 dozer

10. D9 C dozer
11. 10M water tanker
12. Cat. 637 scraper
13. Cat. 666 scraper
14. Cat. 815 compactor
15. Potable air compressor
16. 6" rotary drill
17. 35-ton motor crane
18. 150-ton C. crane
19. Diesel generator
20. D8 dozer
21. Industrial forklift
22. Pickup truck

These data were sought for several reasons. First, information appeared to be readily available. Data concerning value new, estimated salvage values, and probable service lives could be obtained from equipment dealers or the people who are related to the equipment. Data concerning the repair and maintenance costs, and operational characteristics were usually routinely recorded and analyzed by owners or construction contractors. Finally, data which display market evidence were taken from routinely published books as well as from equipment auction firms.

A second reason for these choices was the characteristic similarity of construction and farm equipment with industrial equipment. In other words, the above equipment could be representative of various industrial equipment and their characteristics.

A third reason for pursuing these data was because of an extensive resale market for used equipment. The large number of sales in a given year improved the reliability of this market evidence curve.

The value when new was easily determined for most properties, since the basic assumption is that the value new equals the costs of a property when it was new. This original cost included the total purchase price, any transportation or freight charges, and all installation costs. Also, this original cost should reflect the inflation rate to get the "today's" dollar or the date corresponding to the date of the rest of data.

The salvage value was evidenced by the net cash flow received from the sale of the property upon retirement. Net earnings were equal to gross salvage price less any costs of removing the property. Since the actual salvage value was unknown until the actual point of retirement, an estimated value was used. For most of the equipment, the salvage value was estimated considering the market evidence curve and information from experienced consultants who work in this field. That is, the salvage value was set equal to the value of the market evidence curve at the estimated probable life value and adjusted appropriately.

In the determination of the probable service life, an appropriate Iowa-type curve had to be found. Although the equipment was handled as a unit property, the Iowa-type curves were still needed to compute the probable life values for any age other than zero because, as the unit ages, the probable life increases.

For the actual rate of return, or discount rate, it should be chosen to reflect a reasonable and inflation free rate of return for the company rather than the erroneous reflection of the company's financial policy or prevalent economic conditions at the time of the valuation. As a reasonable approximation of an inflation free rate of return, an annual rate of 7 percent was used for this dissertation [17].

1. Repair and maintenance costs data

Repair and maintenance costs are one of the main factors which affect the operation returns of that equipment. Thus, the collection of this data is necessary for this analysis.

Most of the repair and maintenance cost data of farm equipment was selected from the Hunt's paper [18] titled "Eight Years of Farm Machinery Cost Monitoring." These data are shown in Figure 6.1. The figures represent the relationship between ratio of repair and maintenance costs against value new and accumulated services used. These graphs show the slow increase in the early ages and rapid growth in the later ages. However, the data needed are the R and M costs (repair and maintenance costs) per the unit period of time or age. These collected data, represented by the accumulated unit period of time or service, are needed to translate to the data per unit period of time. The procedures are represented in Table 6.2 for gas tractor as an example. In these procedures, the average usages per year of each piece of equipment are collected for further calculation. The data of average usage per year are shown in Table 6.1 for each piece of equipment. These average usage data were obtained from the Journal of American

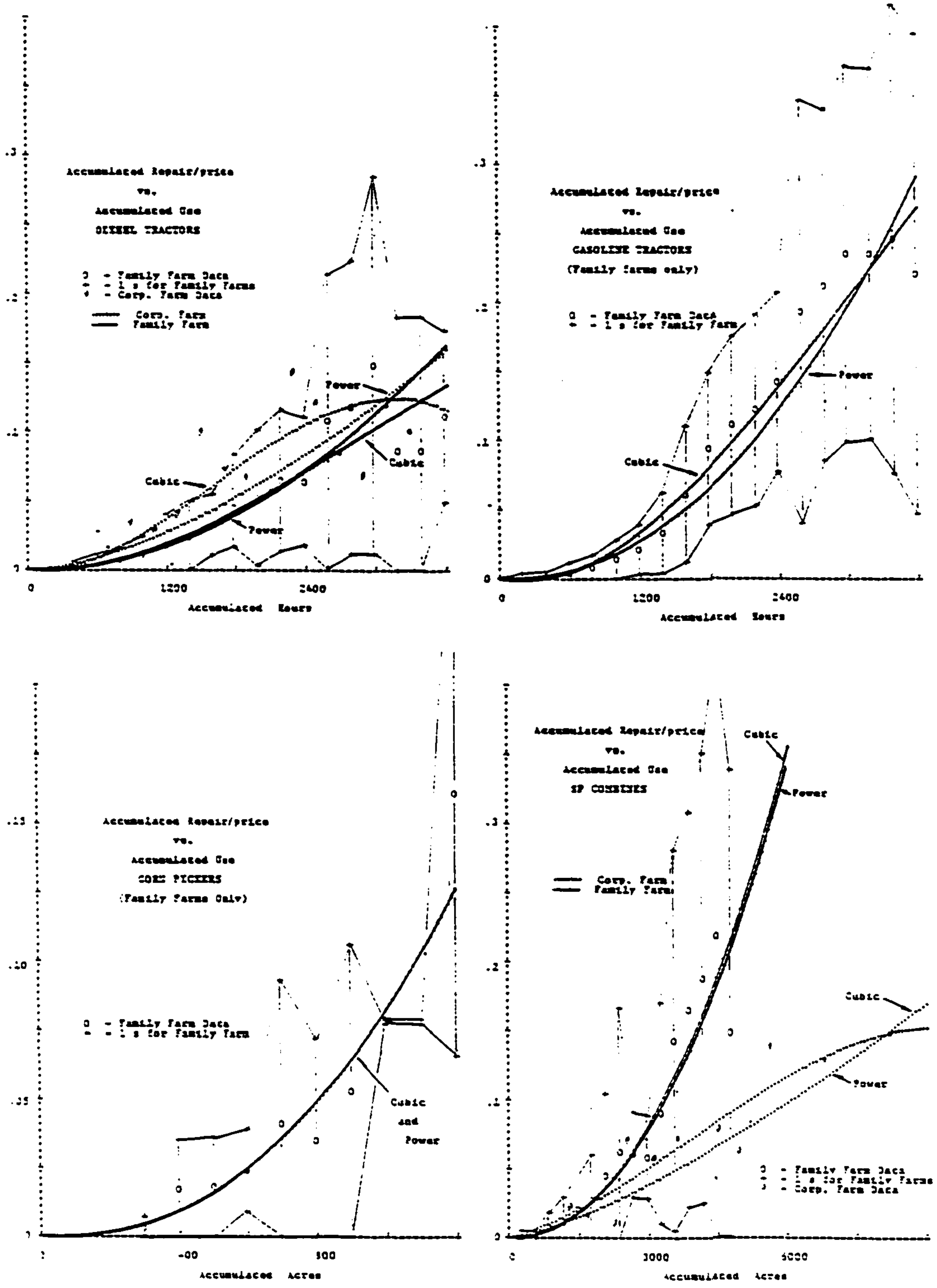

Figure 6.1. Accumulated R and M costs for farm equipment [18]

Society of Agricultural Engineers [19] and Hunt's paper [18]. Finally, adjustment of average usage data was made by the discussions with an agricultural engineer [20] and a consultant [21].

In the case of construction equipment, which are numbered from 7 to 19 of collected equipment, the data of R and M costs were derived from the record of the Green International Construction Company [22]. In the records of the Green Company, the R and M costs were divided into two sections: labor costs and parts costs. The calculation for the R and M costs of the unit period of time (year) was done in the same way as was the farm equipment. The procedure for the combination of labor costs and parts costs represented in Tables 6.3, 6.4, and 6.5. Also, the data of average usage per year, value new, and hourly wages were obtained from the contractors' equipment cost guide [23]. These data are necessary for the calculation of R and M costs of equipment. Table 6.1 shows these data. In the calculation of parts, the inflation rate was considered. The average inflation rate of the past 20 years has been about 7% per year [24]. Since there was one year and four months difference between the date of issue of the contractors' equipment cost guide [23] and that of reports for R and M costs of the Green Company, 9% of inflation was accepted for the calculation. Therefore, the inflation-free observed time is 1974 for farm equipment, 1984 for construction equipment, and 1978 for the other equipment which were obtained from Whelan's thesis and adjusted with new information.

The repair and maintenance costs data are shown in Appendix A. These figures were divided by the value new of each equipment, so that

Table 6.1. Data needed for the calculation of R&M costs

Equipment	Avg. Usage/Yr. Hours (Acres)	Value New ($) (Standard Year)	Hourly Wage ($)
Gas Tractor	500	8093 (74)	-
Dis Tractor	500	8505 (74)	-
Combine	(600)	16365 (74)	-
Corn Picker	(250)	2084 (74)	-
F. Harvester	(200)	4042 (74)	-
Hay Baler	80	2552 (74)	-
35-Ton Truck	1450	289700 (84)	17
50-Ton Truck	1450	384500 (84)	17
D9 Dozer	1450	283820 (84)	16.5
D9 c Dozer	1450	372610 (84)	16.5
10M W. Wagon	1300	348700 (84)	17
637 Scraper	1520	473200 (84)	16.5
666 Scraper	1520	359630 (84)	16.5
Cat 825 Compactor	1200	142470 (84)	16.5
900 Air Comp.	1480	56500 (84)	16.5
6-in. Rot. Drill	800	76895 (84)	16.5
35 M. Crane	1330	241540 (84)	17
150 C. Crane	1460	553600 (84)	17
Dis Generator	1100	18422 (84)	17
D8 Dozer	1400	165000 (78)	-
Ind. Forklift	2000	18350 (78)	-
Pickup Truck	1800	5450 (78)	-

Table 6.2. Example of the calculation for R and M costs per year

Gas Tractor

Cum. hrs	Cum. R and M costs/V_N	R and M/V_N of periodic 400 hrs	R and M/V_N of periodic 500 hrs (1 yrs)	Age
400	0.001	0.001		
800	0.011	0.01	0.0035	1
1200	0.032	0.021	0.018	2
1600	0.064	0.032	0.0345	3
2000	0.1	0.036	0.044	4
2400	0.14	0.104	0.051	5
2800	0.184	0.044	0.055	6
3200	0.228	0.044	0.0565	7
3600	0.274	0.046		

Table 6.3. Labor costs for D9 Dozer

Hours Range	Repair Hrs./ Machine Hrs.	Age	Yearly (1450 Hrs.) Hours Range	Repair Hrs./ Yr.	Times Hourly Wage ($16.50)
0 - 2000	0.088	1	0 - 1450	127.6	2105
2000 - 4000	0.154	2	1450 - 2900	187	3086
4000 - 6000	0.205	3	2900 - 4350	241.2	3979
6000 - 8000	0.265	4	4350 - 5800	297.3	4905
8000 - 10000	0.298	5	5800 - 7250	372.3	6142
10000 - 12000	0.310	6	7250 - 8700	407.4	6721
		7	8700 - 10150	433.9	7159
		8	10150 - 11600	449.5	7417

Table 6.4. Parts costs for D9 Dozer

Hours Range	Costs ($)/ Machine Hr.	Times Inflation Factor (9% for 1.25 Yrs.)	Age	Yearly (1450 Hrs.) Hours Range	Cost ($)/ Yr.
0 - 2000	8.478	9.241	1	0 - 1450	13399
2000 - 4000	10.264	11.188	2	1450 - 2900	15152
4000 - 6000	18.833	20.528	3	2900 - 4350	19492
6000 - 8000	30.049	32.753	4	4350 - 5800	29766
8000 - 10000	33.066	36.042	5	5800 - 7250	45047
10000 - 12000	17.036	18.569	6	7250 - 8700	49794
			7	8700 - 10150	49640
			8	10150 - 11600	26925

Table 6.5. Total R and M costs for D9 Dozer

Age	Labor Cost ($)	Parts Cost ($)	Total Cost	Total Cost/V_N
1	2105	13399	15504	0.0546
2	3086	15152	18238	0.0827
3	3979	19492	23471	0.0827
4	4905	29766	34671	0.1222
5	6142	45047	51189	0.1804
6	6721	49794	56515	0.1991
7	7159	49640	56799	0.2001
8	7417	26925	34342	0.1210

Table 6.6. Final repair and maintenance costs per value new of each age

Equip. Age	Gas Tractor	Diesel Tractor	S.P. Combine	Corn Picker	Forage Harvester	Hay Baler
1	0.0035	0.0025	0.003	0.003	0.018	0.017
2	0.018	0.009	0.007	0.012	0.034	0.0431
3	0.0345	0.01725	0.017	0.0265	0.043	0.0593
4	0.044	0.02125	0.024	0.0405	0.046	0.066
5	0.051	0.02625	0.028	0.076	0.052	0.0747
6	0.055	0.02625	0.040		0.05	0.0693
7	0.565		0.045		0.059	0.0717
8			0.053		0.056	0.0726
9			0.067			
10						

Table 6.6. (continued)

Equip. Age	35-Ton Truck	50-Ton Truck	D9 Dozer	D9 C Dozer	10M Water Tanker	Cat 637 Scraper
1	0.1056	0.0217	0.0546	0.0497	0.0389	0.0892
2	0.0571	0.0173	0.0643	0.0678	0.0468	0.0856
3	0.0285	0.0192	0.0827	0.0862	0.0560	0.0945
4	0.0316	0.0339	0.1222	0.1092	0.0712	0.1144
5	0.0860	0.0658	0.1804	0.1424	0.0779	0.1458
6	0.1094	0.0896	0.1991	0.1658	0.0866	0.1390
7	0.1157	0.1120	0.2001	0.1877	0.0919	0.1270
8		0.1327	0.1210	0.12102	0.0908	
9				0.2116		
10				0.1938		

Table 6.6. (continued)

Equip. Age	Cat 666 Scraper	Cat 825 Compactor	900 Air Loss	6" Rotary Drill	35 ton M. Crane
1	0.0609	0.0737	0.0414	0.0737	0.0296
2	0.0609	0.0794	0.0770	0.0804	0.0070
3	0.1401	0.0908	0.0955	0.0761	0.0256
4	0.1664	0.0954	0.0935	0.0725	0.0704
5	0.2060	0.0976	0.0810	0.0822	0.0742
6	0.2577	0.2574	0.1075	0.1116	0.0613
7	0.3288	0.2149	0.1681	0.1395	0.0450
8		0.1299	0.2990	0.1668	0.0447
9				0.1912	
10				0.2105	

Table 6.6. (continued)

Equip. Age	150 ton C. crane	DSL Generator	D8 Dozer	Ind. Forklift	Pickup Truck
1	0.0655	0.0268	0.1258	0.0210	0.1728
2	0.0566	0.0637	0.1314	0.0322	0.2294
3	0.0589	0.1178	0.1523	0.0360	0.2437
4	0.0804	0.1342	0.1619	0.0657	0.1688
5	0.0673	0.0855	0.1629	0.0722	0.1354
6	0.0657	0.0855	0.1937	0.0583	
7			0.2084	0.0755	
8				0.0929	
9				0.0850	
10					

the figures are the ratios of the value new. The reasons for using the ratios are the simplicity of use and the facility of comparisons. Table 6.6 shows that the final R and M costs data for the equipment collected according to the ages of life. In this table, the figures of farm equipment are usually smaller than those of construction equipment. That seems construction equipment works more per unit time and more severely than farm equipment.

The smoothed curves of repair and maintenance costs are shown in Table 6.7. These curve equations were obtained by using the least-sum-of-squares method with the help of the SAS computer program. Usually, these are represented by x and x^2 with the minus sign between them.

Table 6.7. Smoothed curves of repair and maintenance costs data

Equipment	Curve Equation
G. Tractor	$0.012296\ x - 0.000550\ x^2$
D. Tractor	$0.006006\ x - 0.000140\ x^2$
Combine	$0.005641\ x + 0.000123\ x^2$
Corn Picker	$0.004163\ x^2 - 0.000286\ x^3$
F. Harvester	$0.015001\ x - 0.000910\ x^2$
Hay Baler	$0.019071\ x - 0.001078\ x^2$
35 ton truck	$0.016445\ x - 0.000024\ x^2$
50 ton truck	$0.007808\ x + 0.000973\ x^2$
D9 Dozer	$0.028089\ x - 0.000075\ x^3$
D9 C Dozer	$0.032187\ x - 0.000117\ x^3$
10 M W. Tanker	$0.021518\ x - 0.001099\ x^2$
Cat 637 Scraper	$0.040468\ x - 0.002672\ x^2$
Cat 666 Scraper	$0.049399\ x - 0.000165\ x^3$
825 Compactor	$0.028939\ x - 0.001001\ x^2$
900 Air Comp.	$0.022543\ x + 0.000007\ x^3$
6" Rot. Drill	$0.021981\ x - 0.0001622\ x^2$
35 T. M. Crane	$0.013166\ x - 0.000570\ x^2$
150 T. C. Crane	$0.021637\ x - 0.001239\ x^2$
DSL Generator	$0.025669\ x - 0.001323\ x^2$
D8 Dozer	$0.045279\ x - 0.002418\ x^2$
Ind. Forklift	$0.018699\ x - 0.000752\ x^2$
Pickup truck	$0.058844\ x - 0.003569\ x^2$

That shows the normal shape of the smoothed curves for R and M costs. This shape will be used to determine the standard curves of R and M costs later.

2. Service intensity data

The service intensity is one of the important factors in estimating the declining operation returns for this study. Whelan [14] tried to prove the validity of the Elfar model with the R and M costs and down time costs, but this procedure with those factors seems to have difficulty in measuring the actual loss in property value. Thus, the new trial with the R and M costs and service intensity rates is done in this dissertation. As discussed before, there could be many factors which reduce the annual operation returns. These factors can be summarized by three factors: rising operating cost, impaired service quality, and improved alternatives. Here, service intensity seems the only estimated data that are easily obtained and directly influenced by the second and third factors above. Thus, the assumption is that the service intensity rates reduce with the rate of impaired service quality and improved alternatives, and this reduction seems to make the operation returns of equipment to decline with almost the same rate of the reduction. Thus, these service intensity data are one of the most important sets of data to be collected.

The intensity data for farm equipment were collected from a journal [19] and the others were collected from a government publication [25] and Dynamic Equipment Policy [10]. These were summarized in Table 6.8. According to the data collected, farm equipment data show a slower

Table 6.8. Service intensity data

Name Age	Tractor (hrs)	Combine	Corn Picker	Hay Baler	Forage Harvester	Pickup truck (1000 miles)
1	472	222	124	71	84	14.4
2	483	231	120	76	91	15.2
3	495	207	87	76	78	14.9
4	554	157	91	89	53	13.9
5	504	148	71	60	57	12.4
6	449	128	74	65	26	11.7
7	468	99	77	63	77	10.9
8	471	80	76	54	—	10.3
9	485	99	57	51	--	9.6
10	455	68	63	41	68	8.6
11	461	64	48	34	--	8.2
12	422	59	52	41	65	8.3
13	407	66	32	74	--	5.3
14	362	52	52	35	—	5.3
15	382	54	35	31	28	
16	374	38	39	35		
17	349	41	46	41		
18	281	31	50	24		
19	455	30	63	20		
20	284	29	34			

Table 6.8. (continued)

Name Age	Construction Heavy Tractor (hr)	Construction Truck (1,000 miles)
1	650	14.1
2	630	12.8
3	590	11.6
4	575	10.5
5	550	9.4
6	525	8.4
7	500	7.6
8	470	6.9
9	445	6.3
10	420	5.7
11	395	5.2
12	365	4.8
13	345	4.5
14	320	4.3
15	290	4.1
16		
17		
18		
19		
20		

decrease of service intensity than the other types of equipment. The smoothed curves for this service intensity data were obtained by use of least sum of squares method. These are represented in Table 6.9.

In these service intensity data, there was a tendency for heavier equipment to last longer and decline at a slower rate of intensity than light equipment. Though the data for farm equipment were found [19] and analyzed item by item, data for each of the other types of equipment were not found and only the representative service intensity data were obtained [10,25]. These representative service intensity data will be used for all of the other equipment except farm equipment.

3. Market evidence data

Market evidence data are important because the K values of these data are used as criteria to compare with those of R and M costs and service intensity. These market evidence data were obtained from the routinely published valuation guide [26,27] for farm equipment. On the other hand, the other data were derived from auction companies [28], Green Construction Company where they collected the auction data and made a simple table [22], and the valuation guide [29]. Usually, they have two kinds of value for equipment. These are average resale values and average as-is values.

According to the Blue Book [27], the average resale value is a guide as to the probable price a standard "as advertised" model will bring on the open market when sold in single lots, after having been properly exposed to the market, by a dealer usually engaged in the implement business to a purchaser willing and able to buy, assuming the

Table 6.9. Smoothed curve graphs for service intensity

Equipment	Graph Equation
Tractor	$1 - 0.0965689 - .00009793\ x^3$
Combine	$1 + 0.13487819 - 0.11647211\ x + 0.00354272\ x^2$
Corn Picker	$1 + 0.06129428 - 0.09206833\ x + 0.0029727\ x^2$
Hay Baler	$1 + 0.02445473 - 0.05817998\ x + 0.00117404\ x^2$
F. Harvester	$1 - 0.14617940 - 0.02683363\ x$
Pickup Truck	$1 - 0.01110414 - 0.00679704\ x^2 + 0.00026015\ x^3$
Construction Heavy Tractor	$1 + 0.03916597 - 0.044736842\ x$
Construction Truck	$1 + 0.09784117 - 0.10183356\ x + 0.00322060\ x^2$

sale is an "arm's length" transaction and that neither party is under duress to buy or sell. According to the Official Guide [26], average resale values are applicable to machines with average amount of reconditioning made by the reporting dealers.

The average "as-is" value of the Blue Book [27] is expressed as follows. These figures represent the average value of tractors and farm machines in usable as-is condition prevailing throughout the country. Within this general group of values would fall trade-in allowance by dealers, direct sales between farmers, and equipment sold at auction. In the official guide, the average as-is values are in all instances applicable to machines with standard or regular equipment and rubber tires except where otherwise noted. A deduction from these values must be made by the dealer for machines on which any of the standard or regular equipment is missing. This average as-is value is computed by subtracting average reconditioning costs from average resale values, then deducting 15% from that difference. Thus, the resale values seem to be close to buyer's prices and the as-is values are seller's prices.

As criteria in this dissertation, the resale values are used and collected since the values of average, dependable, well-conditioned machines and the auction sale values are close to the resale values. Furthermore, the as-is value is usually the bottom price and the real value to the owner is always greater than that. The average as-is values can be easily calculated as around four-fifths of the average resale values according to the reference books mentioned above.

These collected data are shown in Table 6.10. It is also

Table 6.10. Market evidence data

Equipment Type	Value New	Age	1	2	3	4	5	6	7	8	9	10
Gas Tractor	7620	PCT	84.3	77.6	71.4	65.7	60.4	55.6	51.1	47.0		
Diesel Tractor	8032	PCT	86.3	79.4	73.0	67.2	61.8	56.8	52.3	48.1		
S.P combine	16350	PCT	68.5	62.0	56.1	50.8	45.9	41.6	37.6	34.0	30.8	27.9
Corn Picker	2084	PCT	66.8	58.8	51.7	45.5	40.0	35.2	31.0	27.2	23.9	21.0
Hay Baler	3303	PCT	67.4	61.0	55.6	49.9	45.1	40.8	36.9	33.4	30.2	
F. Harvester	2552	PCT	68.9	61.4	54.6	48.6	43.2	38.5	34.2	30.5	25.1	20.7
35 ton End dump truck	289,700	PCT	64.8	54.6	47.2	43.8	41.8	32.4	26.4	24.0	18.0	12.2
50 ton End dump truck	384,500	PCT	64.8	54.6	47.2	43.8	41.8	32.4	26.4	24.0	18.0	13.2
D9 dozer	283,820	PCT	78.0	66.0	60.0	50.4	42.0	36.0	33.6	30.0	24.0	18.0
D9 C dozer	372,610	PCT	78.0	66.0	60.0	50.4	42.0	36.0	33.6	30.0	24.0	18.0
10 M Gal Water Wagon	348,700	PCT	80.0	67.8	52.2	37.8	36.0	27.0	24.0	20.4	16.8	13.2
CAT 637 Scraper	473,200	PCT	78.0	69.6	62.4	46.8	38.4	31.2	29.4	24.0	21.6	18.0
CAT 666 Scraper	359,630	PCT	68.4	56.4	44.4	34.8	25.2	18.0	15.6	14.4	13.2	12.0

CAT 825 Compactor	142,470	PCT	75.6	64.8	58.8	54.0	45.6	42.0	37.8	32.4	22.8	18
900 Portable Air Compressor	56,500	PCT	75.6	66.0	50.6	36.0	24.0	16.8	10.8			
6" Rotary Drill	76,895	PCT	78.0	66.0	56.4	46.8	42.0	36.0	30.0	28.8	26.4	24.0
35 Ton Motor Crane	241,540	PCT	82.8	70.8	62.4	54.6	44.4	38.4	33.6	30.0	18.0	12.0
150 Ton Crawler Crane	553,600	PCT	84.0	73.2	63.6	55.2	48.0	44.4	40.8	37.2	33.6	31.2
90 KW Diesel Generator	18422	PCT	84.0	75.6	72.0	66.0	63.6	60.0	56.4	52.8	46.8	42.0
D8 dozer	165,000	PCT	82.2	73.8	65.4	56.4	51.0	44.4	42.0	39.0	33.6	28.8
Industrial Forklift	18,350	PCT	81.9	75.6	65.3	54.1	40.7	35.8	33.3	31.2	27.0	23.7
Pickup Truck	5,450	PCT	80.4	67.3	54.4	46.2	35.5	28.4	22.5			

represented as a percent of its value new. Since the times of collected R and M cost data are different, the value new data are collected according to the year of R and M costs data collected. Thus, the value new of farm equipment, based on 1974 data, D8 dozer, Industrial forklift, and pickup truck are based on 1978, and the other construction equipment is derived from 1984 data, and the costs data are consistent with the year of the value new of each equipment.

4. Probable life and salvage value data

These data are necessary to determine the K values of the market evidence data and the K values of the Delta method with the R and M cost data and service intensity data. Probable service life can often be estimated with a reasonable degree of accuracy based on lifespans of similar units and various life analysis techniques. However, no life analysis studies for this equipment were found. Therefore, the probable service lives for farm equipment was estimated based on reference books [30,31,32] and discussion with experienced people [20,21]. The probable service lives of the other equipment were estimated based on the cost guide [23] and industry experience [22]. The salvage values were estimated based on the same procedure as probable service lives. These are shown in Table 6.11.

Most of the construction equipment lifespans were 10 years and usually the probable service lives of farm equipment were longer than construction equipment since construction equipment works hard and severe compared with farm equipment. Also, the service lives of heavy equipment have a tendency of longer life. Usually, the economic life is shorter and the salvage value is bigger than the probable lives and salvage values shown in Table 6.16. Thus, there is an inverse relationship between probable lives and salvage values.

Selection of the most appropriate Iowa type curve was made by the trial and error curve fitting procedure. Based on this procedure, the L5, S6, and R5 curves were selected for most of the equipment as giving the best fit of the computed value curve to the market evidence data.

Further analysis made by Whelan [14], revealed that, even though the degree of fit between the computed curve and the market evidence data were better for some curve types than for others, the best fit for each curve always resulted in the selection of the same value of T. Namely, the Iowa type curve selection affected the closeness of it, but made no apparent difference in the T-factor value selected as being the best estimate.

B. Data for Group Property

Procedures previously presented are either based on or biased toward unit properties. However, group properties are also important since a lot of portion of industrial properties is the vintage groups. Although the application of the proposed procedures to group properties is more complicated than unit properties, the basis of procedure and validity of results are unchanged. The significant differences, however, will be explained and the data for group properties will be presented.

Table 6.11. Probable life and salvage value

Equipment	Estimated (yr) Probable Life	Salvage Percent Per Value New
Gas tractor	15	10
Diesel tractor	17	15
combine	12	15
Corn Picker	10	20
F. Harvester	10	20
Hay Baler	12	15
35 ton truck	10	12
50 ton truck	10	12
D9 dozer	10	18
D9C dozer	10	18
10M water wagon	10	12
637 scraper	10	18
666 scraper	10	12
CAT 825 compactor	10	18
900 Air compressor	7	10
6" Rotary drill	12	15
35 ton M. Crane	10	12
150 ton C. crane	15	10
Diesel generator	15	20
D8 dozer	15	15
Industrial forklift	10	25
Pickup truck	13	0

1. Group property considerations

The consideration of group properties as opposed to unit properties is another important subject in the valuation of industrial properties. Group property can be either a group of identical pieces of equipment or a group of un-identical equipment which make an operation unit for common functions or objectives like assembly lines, oil refineries, etc. The difference between unit property and group property is in the consideration of frequency curves of retirement.

The procedure for handling group property is called "unit-summation procedure" or "equal life group procedure." The basic concept of this procedure is to separate the surviving units comprising the group into frequency groups of units of like probable life as predicted from a forecasted retirement dispersion pattern like the Iowa type curves. Since the units of a frequency group have the same expected life, they can be treated in total as a single item having that life.

The modified condition percent factor for all the survivors of the group at a given age is then obtained by weighting each modified condition percent factor, calculated on a unit basis for each frequency group surviving, by the number of units in each frequency group. Then, the weighted average modified condition percent factor of the entire group property is obtained. Figure 6.2 shows the S0 Iowa type curve with a 5 year average service life and how the survivors are segregated into frequency groups with the increasing probable lives as frequency group ages.

Furthermore, each of the segregated frequency groups can be

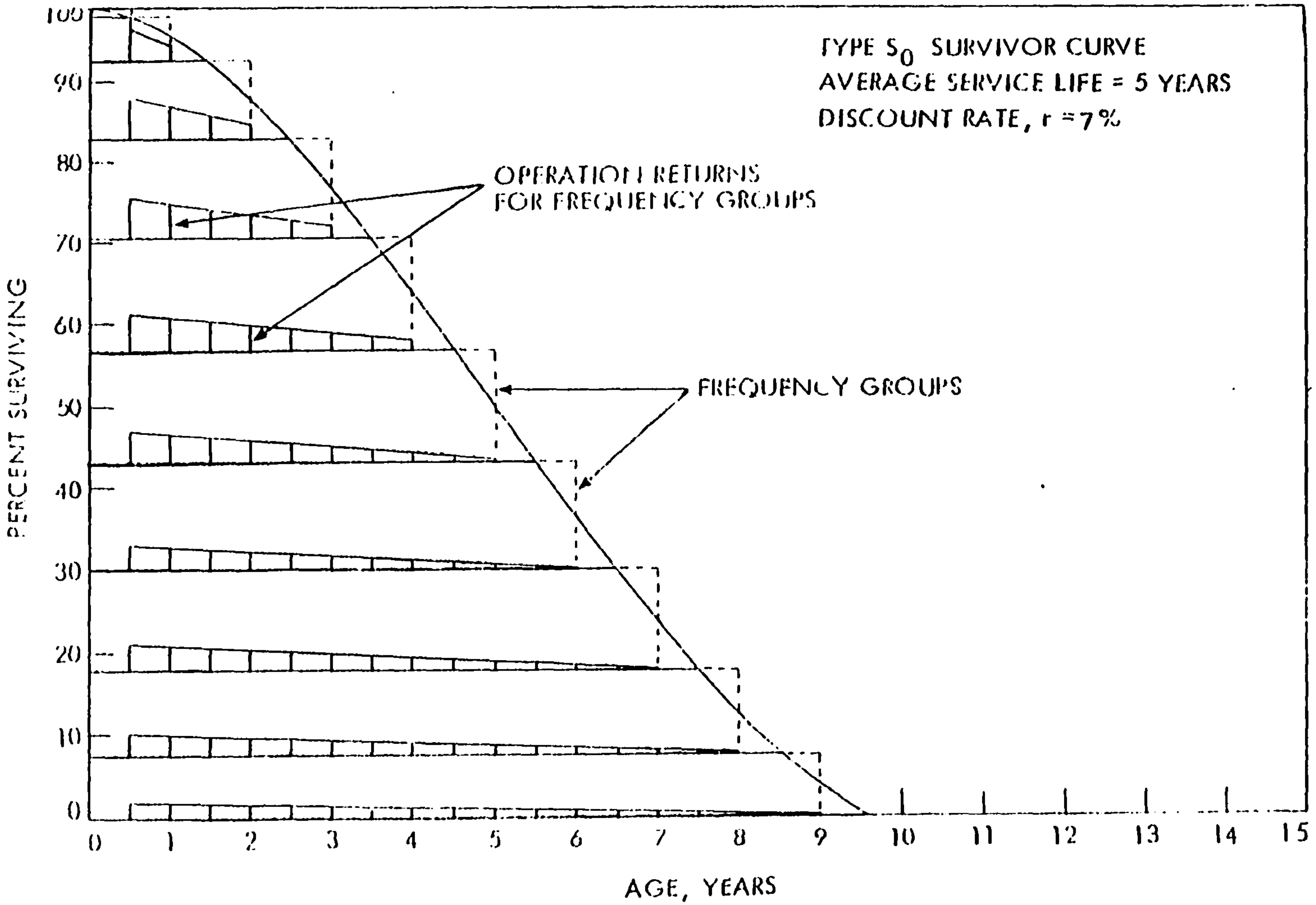

Figure 6.2. Application of unit summation procedure to declining operation returns case [12]

considered as a unit property of unique average service life. For example, the average service life of the first frequency group at the top of Figure 6.2 is 1 year while that of the last frequency group is 9 years. Thus, they have different average service lives and the assumed operation returns of the first age for each frequency group would be proportional to the size of each frequency group. Thus, for the unit property of each frequency group, the decrease in operation returns would be different each case due to the different average service lives and salvage ratios. That means that the K values of each frequency group are different. Therefore, the K values of an overall property of a group seem to be the weighed average of the K values of each frequency group. Consequently, the weighed average of K values which are obtained by using the procedures for unit property would be the K values of the group property in question.

2. Data collected

In the case of group property, data for the service intensity and the repair maintenance costs as the factors of declining operation returns of unit property were not easy to obtain because of the difficulty of record keeping for a particular group of properties. Thus, as the substitutes for the decrease of service intensity and the increasing R and M costs, the decrease in gross operating revenue and the increasing operating costs were assumed to be the main factors of declining operation returns in group property. The decrease in gross operating revenue is thought to be caused by the decreasing service intensity which might be affected by lowered efficiency and quality, downtime, obsolescence, etc. The increasing operating costs is mostly due to the increasing repair and maintenance costs caused by the increasing usage of power, labor, material, parts, etc. These two main factors of group property were treated in the same way as those of unit property in the Delta method. In Figure 5.1, the f(x) is the decreasing gross operating revenue and t(x) is the increasing operating costs in case of group property.

The data of two oil refineries as group properties were collected [15] and used in the experiment. Sufficient data were available for one of the refineries to allow an analysis at three different ages. Thus, four sets of data of two refineries were in effect available for analysis. In order to ensure anonymity, the four sets of data will hereafter be referred to as 1) Alpha-1, 2) Alpha-2, 3) Alpha-3, and 4) Beta refineries. The three sets of data for "Alpha" refinery were obtained from the same refinery but collected at three different ages, i.e., 5, 10, and 16 years old. The data for "Beta" refinery were obtained at the age of 10. The Income statements of these refineries are shown in Table 6.12. The operating costs statements for these refineries are also represented in Table 6.13.

The decreasing gross operating revenue which is assumed to play the same role as the declining service intensity of unit basis is represented as differences with the gross operating revenue of the modern replacement. These differences for Alpha-1, -2, -3, and Beta refineries are 2.9, 5.1, 7.5, and 12.9, respectively. The units of the figures are also $MM/YR. The average life of the Alpha and Beta

Table 6.12. Income statements for Alpha-1, -2, -3, and Beta refineries vs. modern replacement[a]

Refinery	Alpha				Beta	
	Modern Replacement	Alpha-1 (5 yrs old)	Alpha-2 (10 yrs old)	Alpha-3 (16 yrs old)	Modern Replacement	Beta (10 yrs old)
Taxable income	13.2	12.0	9.9	8.4	21.0	5.2
Tax (federal)	6.4	5.8	4.8	4.0	10.1	2.5
Net income	6.9	6.9	5.1	4.4	10.9	2.9
Depreciation	7.8	5.8	3.8	1.8	12.3	8.3
Cash income after tax	14.7	12.0	8.9	6.2	23.2	11.0

[a] All figures in units of $MM/YR.

Table 6.13. Operating costs statements for Alpha-1, -2, -3, and Beta refineries vs. modern replacement[a]

Refinery	Alpha				Beta	
Costs	Modern Replacement	Alpha-1 (5 yrs old)	Alpha-2 (10 yrs old)	Alpha-3 (16 yrs old)	Modern Replacement	Beta (10 yrs old)
Labor	6.8	7.4	9.4	10.8	16.6	22.8
Material	1.5	1.8	2.2	2.7	5.7	8.5
Outside fuel	0.4	0.4	0.4	0.4	6.5	7.2
Chemicals	3.8	3.9	4.0	4.2	5.7	5.7
Utilities	1.2	1.2	1.3	1.3	2.8	3.1
Total	13.7	14.7	17.3	19.4	37.3	47.3

[a] All figures in units of $MM/YR.

refineries is 20 years. The replacement costs for the Alpha and Beta refineries is reported as 150 million dollars and 246 million dollars, respectively. The salvage values are around 5 percent of the replacement costs.

The above collected data will be used to find the K values for each refinery. The K values will be obtained by using the ratio method and the Delta method. Since the group property like a refinery can provide the income statement, that is, operation returns for some ages, the operation returns can be directly fit to the basic model which describes the relationship between R_1 and R_x. For this direct fitting, the ratio method will be used. The data for operating costs, operating revenues, value new, salvage ratio, and average life will be used for the Delta method in the same way as unit property. These K values from both the ratio method and Delta method will be compared.

VII. RESULTS AND DISCUSSION

Using the data collected and procedures developed, K values for a number of properties were estimated. The K values derived from the only R and M data, R and M plus intensity data, and market evidence data were compared and determined the most realistic approach to estimate the value of industrial property in addition to the choice of better model. K values for group property were also obtained and discussed. The standard tables of property value at x were developed and presented in this chapter.

A. Estimation of K Values for Market Evidence Data

K values were estimated for all equipment. The computed value curves according to the variation of K values were compared to the corresponding market evidence data using a least sum of squares method to measure the goodness of fit. The K values resulting in the best fit were then selected as a slope factor exhibited by the properties.

1. Comparison of Elfar model and Y model

Depending upon the model used, the goodness of fitting were varied. The comparisons of the V_x of Y model, the V_x of Elfar model, and the V_x from market evidence data are shown in Appendix E. Two of the collected equipment are illustrated as examples in Figures 7.1 and 7.2. These figures showed that the V_x of Elfar model is usually further from that of Y model with the V_x of market evidence data. That means the V_x of the Y model produces a better fit with market evidence data than the V_x of the Elfar model. Usually, the variation of K value and T factor

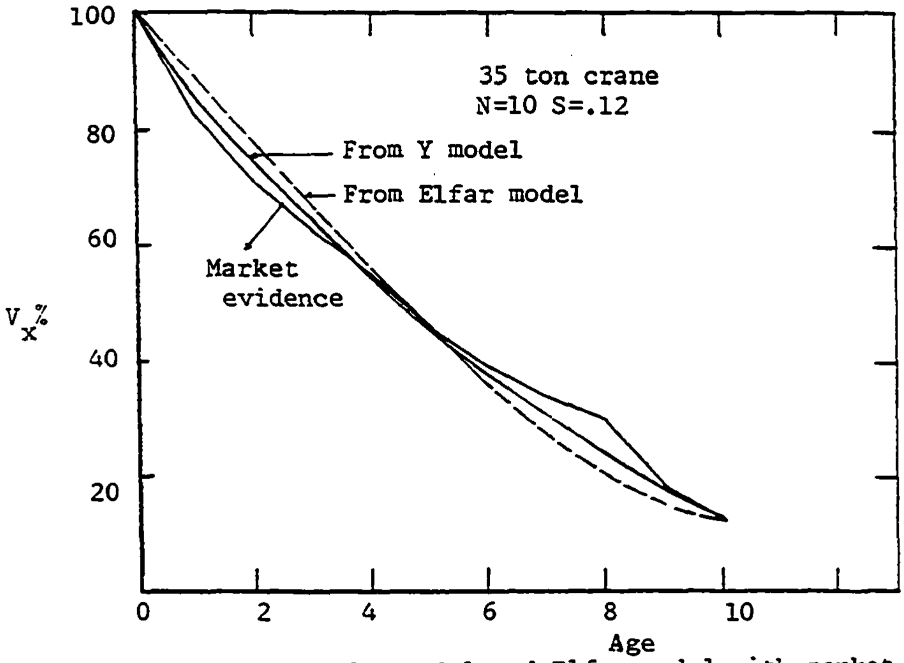

Figure 7.1. Comparison of Y model and Elfar model with market evidence

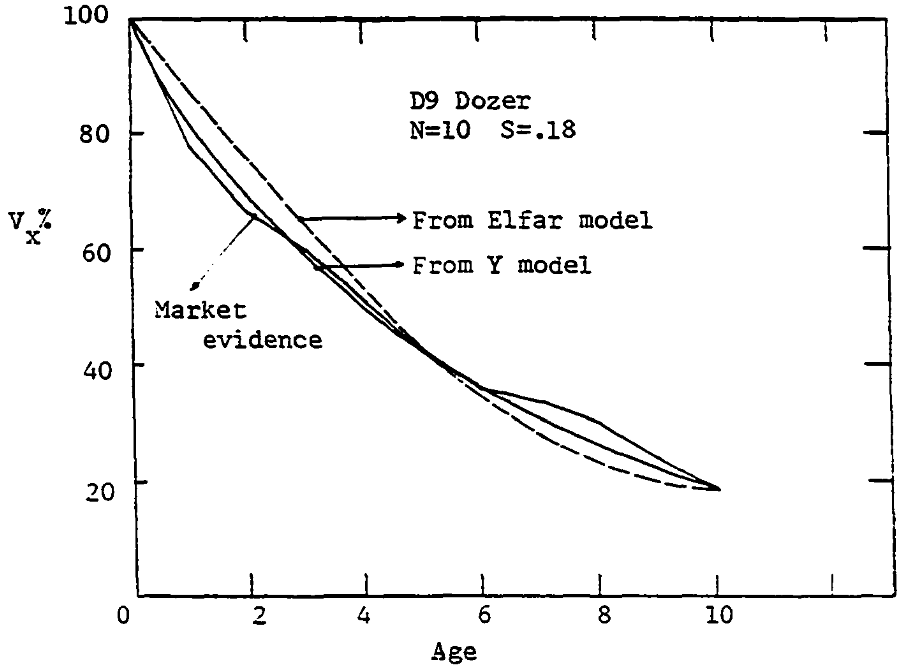

Figure 7.2. Comparison of Y model and Elfar model with market evidence

value can produce different shape of graph in either the Y model or the Elfar model, respectively. However, since the basic characteristics of each model are different and the basic shape of graph, namely the rate of decline at each age, is different from each other, the shapes of V_x of the figures are different from each other after all. Usually, market evidence data have a rapid decline of value in the first year after equipment was purchased. This decline of value appears to be caused by a dealer's large sales profit, the tendency of people to buy a new product, freight and installation charges at the beginning, substantial obsolescence in the first year, investment tax credit for the new product or so many unknown factors; however, the Elfar model did not represent rapid decline in the first year and slow decline in subsequent years. Thus, the Y model seems superior to the Elfar model. Afterwards, all the analyses in this study were done by using the Y model instead of the Elfar model.

Thus, the K values were estimated through all the experimental procedures described in the previous chapter with analyzed data of every industrial property. The fundamental data sets were market evidence data, R and M costs data, service intensity rate data, probable life and salvage value for every one of the items. For all these analyses, the rate of return was selected to be 7 percent per year [17] as the most reasonable value for an inflation free annual rate of return. Moreover, according to the analysis of Chapter IV, Figure 4.8, the rate of return term had little effect on the completed equipment values as long as a reasonably accurate rate was used.

2. K values of market evidence

The K values were obtained by using least sum of squares method with collected market evidence data. These results where shown in Table 7.1. The error bound of these K values was around ± .05. However, the values in Table 7.1 were usually the best.

B. Estimation of K Value Using Delta Method

Estimation of K values using the Delta method required collection of data related to the components of depreciation. The components of normally increasing depreciation with the passage of time commonly include the following:

1. rising repair and maintenance costs as parts wear out or fail in service, and
2. decreasing service intensity due to increasing downtime, falling production rate, and increasing functional or economic obsolescence.

The R and M costs data were derived from field reports of time and costs. A regression analysis and time adjustment were applied to the field date during the summation process. The results were plotted as constant dollar, smoothed curves. These accumulated data were translated to the data for unit ages by calculating with annual usage data. Then, the data for unit were applied to the standard curve of the Delta method after appropriate calculations to make same unit with the standard curves. After that, the K values were derived by matching or fitting the Delta values with the standard curves. These fittings were also done by visual fitting based on least sum of squares

Table 7.1. K values of market evidence

Equipment	K value
Gas tractor	0.85
Dis. tractor	0.9
Combine	0.5
Corn picker	0.3
F. Harvester	0.5
Hay baler	0.45
35 ton truck	0.4
50 ton truck	0.4
D9 dozer	0.45
D9 C dozer	0.45
10M w. wagon	0.45
637 scraper	0.45
666 scraper	0.3
825 compactor	0.5
900 air comp.	0.65
6" Rot. drill	0.32
35 M crane	0.8
150 C crane	0.45
Dis generator	0.8
D8 dozer	0.45
I. forklift	0.5
Pickup truck	0.3

method. The standard tables of Delta values with the variety of K values, average lives, and salvage ratios are represented in Appendix B.

1. K values with R and M costs and service intensity

The service intensity rate was assumed in this dissertation as one of the main reasons for declining operation returns in addition to R and M costs. Thus, K values were derived with the consideration of both R and M costs and service intensity rates. Smoothed curves of these data and the composition of these two smoothed curves were the main procedures to obtain the value, $(P_x-P_1)/V_N$. This value was compared to the Delta value of standard curves which were derived by using the Y model.

The examples for estimation of the K values using standard sets of curves according to the determined probable service lives and salvage ratios of each piece of equipment were shown in Figures 7.3 and 7.4. The K values with R and M costs and service intensity rate are summarized in Table 7.2.

2. K values with R and M costs

This analysis method was developed by Whelan [14]. In this section, the K values were derived by using only R and M costs with numbers of real world data. These K values will be used for the comparison with those of R and M costs and intensity rate and those of real market evidence data. The procedure to find the K values was explained in Chapter V. The only difference is to set the decline of the intensity rate as zero, namely, f(x) equals 1. Table 7.3 shows the

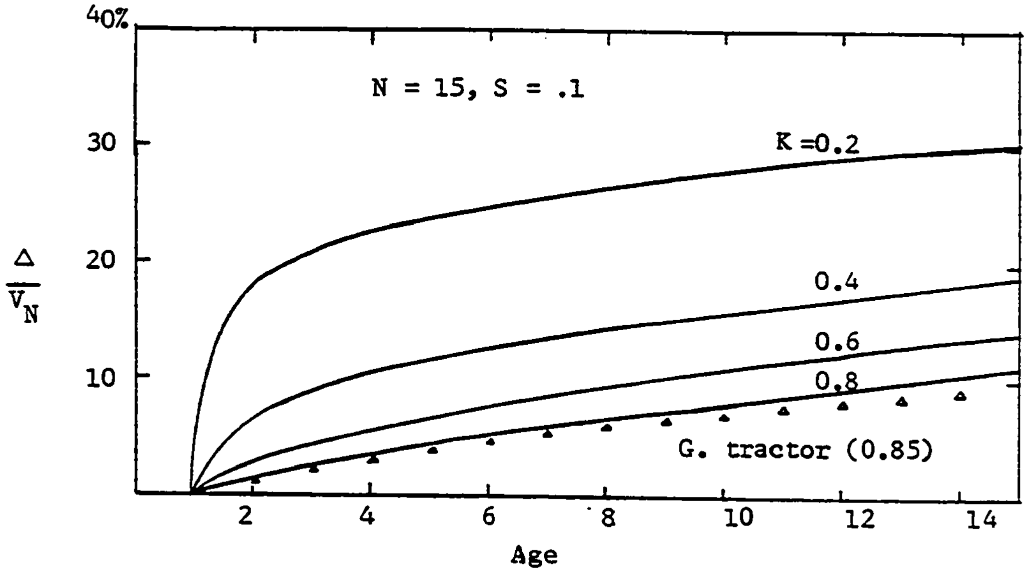

Figure 7.3. Standard curves for Delta method and fitting

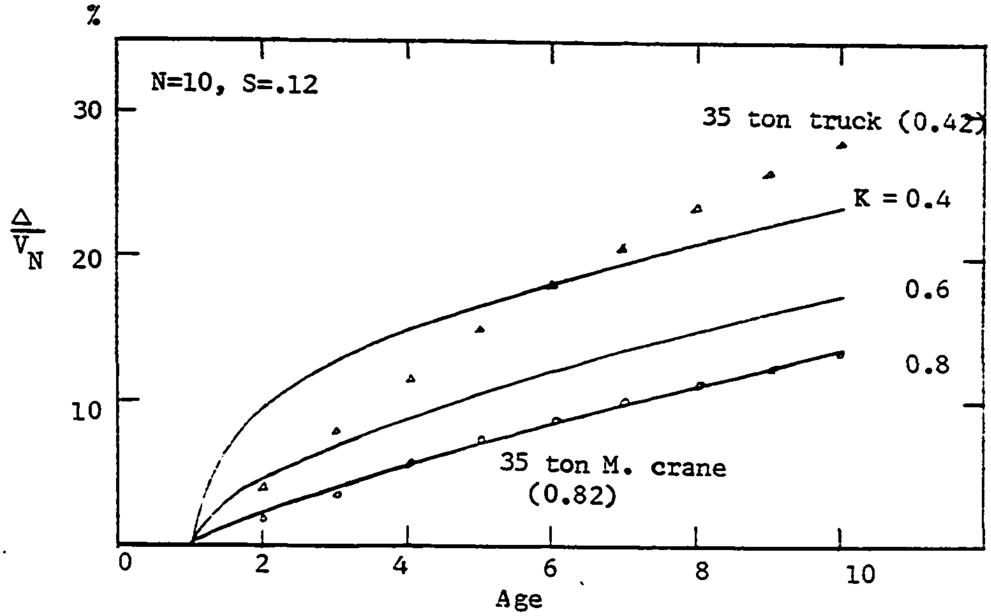

Figure 7.4. Standard curves for Delta method and fitting

Table 7.2. K values with R and M costs and intensity rate

Equipment	K value
Gas tractor	0.85
Dis. tractor	0.90
Combine	0.48
Corn picker	0.50
F. Harvester	0.81
Hay baler	0.58
35 ton truck	0.42
50 ton truck	0.45
D9 dozer	0.40
D9 C dozer	0.40
10M w. wagon	0.47
637 scraper	0.45
666 scraper	0.28
825 compactor	0.50
900 air comp.	0.78
6" Rot. drill	0.38
35 M crane	0.82
150 C crane	0.44
Dis generator	0.68
D8 dozer	0.40
I. forklift	0.62
Pickup truck	0.33

K values with the data of R and M costs. The error boundary for these values is within ± .1.

C. Comparisons of K Values out of R and M, R and M and Intensity and Market Evidence

Three sets of K values were finally obtained and compared. The first one was derived from the market evidence data. The second one was made with the R and M costs data and service intensity rate data through the Delta method. The last one was similar to the second one but using only R and M costs data without service intensity rate. These three sets of K values are summarized in Table 7.4.

The comparison of the three sets of K values revealed several characteristics. First of all, the K values of R and M costs and intensity were always smaller than those of R and M costs. The difference in range of these two sets of K values was from 0.2 to 0.5. These differences were supposed to be in same ratio with the steepness of the intensity rate decrease. But, owing to the model characteristics, the differences between the K values do not represent the same differences on the intensity graph. The smaller the figure of K value becomes, the smaller the change in K value becomes for the same change of the intensity rate. Thus, the same difference of K value in Table 7.4 does not mean the same rate of gap in the intensity rates.

The second observation is that the K values of R and M costs are always bigger than those of market evidence. That indicates that there should be other factors which reduce the operation return with age in addition to the R and M costs.

Table 7.3. K values with R and M costs

Equipment	K value
Gas tractor	1.1
Dis. tractor	1.2
Combine	1.1
Corn picker	.8
F. Harvester	1.2
Hay baler	1.0
35 ton truck	.9
50 ton truck	.9
D9 dozer	.6
D9 C dozer	.6
10M w. wagon	.9
637 scraper	.7
666 scraper	.4
825 compactor	.7
900 air comp.	1.0
6" Rot. drill	.5
35 M crane	1.1
150 C crane	.9
Dis generator	1.0
D8 dozer	.6
I. forklift	.9
Pickup truck	.5

Table 7.4. Comparison of K values

Equipment	K value of market	K value of R and M and Int.	K value of R and M
Gas tractor	0.85	0.85	1.1
Dis. tractor	0.90	0.90	1.2
Combine	0.50	0.48	1.1
Corn picker	0.30	0.50	0.8
F. Harvester	0.50	0.81	1.2
Hay baler	0.45	0.58	1.0
35 ton truck	0.40	0.42	0.9
50 ton truck	0.40	0.45	0.9
D9 dozer	0.45	0.40	0.6
D9C dozer	0.45	0.40	0.6
10M W. wagon	0.45	0.47	0.9
637 scraper	0.45	0.45	0.7
666 scraper	0.3	0.28	0.4
825 compactor	0.5	0.50	0.7
900 Air Comp.	0.65	0.78	1.0
6" Rot. drill	0.32	0.38	0.5
35 M. crane	0.8	0.82	1.1
150 C. crane	0.45	0.44	0.9
Dis. generator	0.8	0.68	1.0
D8 dozer	0.45	0.40	0.6
Ind. forklift	0.5	0.62	0.9
Pickup truck	0.3	0.33	0.5

The third observation is that the K values out of R and M costs and intensity were always closer to those of market evidence than those of R and M costs. Furthermore, the difference between K values of R and M costs with intensity and those of market evidence were quite small; that is, the gap of K values of 16 equipments out of 22 were within 0.1. That means the market values obtained by using the K values out of R and M costs and intensity fit quite close to the actual market evidence values. Thus, the proposed Delta method using R and M costs and intensity rate were shown to be reliable and valid. Therefore, these three observations above indicate that the proposed Y model,

$$R_x/R_1 = 1 - (\frac{.7(x-1)}{N})^T$$

with the condition, $R_x/R_1 = 0$ at $X = N + 1$, represents a realistic relationship between operation returns and market values according to age of the equipment. Also, the Delta method using R and M costs and service intensity rate was shown to give better estimation of K than the Delta method using only R and M costs. Furthermore, the decline of service intensity rates with the age of the equipment might represent and include the whole other factors except R and M costs factor which cause the decline of operation returns of industrial property.

However, the role of experienced judgment in all the procedures is also recommended. The experienced judgment in the collection, interpretation, and utilization of the input data will lead to more reliable results.

Since the proposed Y model and Delta method were proved to be valid and reliable, if R and M costs and service intensity rates were provided every year, one can determine the market value at age x of an industrial property by using Y model and Delta method. Since most of the industrial properties do not keep the market values with their ages due to the scarcity of records of sale the procedures of this dissertation could make a good standard or criterion for the values of them. However, to keep track of all the steps for the determination of value is sometimes a lot of work. To avoid this inconvenience, the concept of standard tables for value at age x was induced. Detailed concepts and explanations for the standard tables will be discussed in a later section.

D. Comparison of K Values of Group Property

Estimation of K values of group properties using the ratio method was obtained from the income statement data. The procedure and K values were shown in Table 7.5. The best fitting K values were obtained by the trial and error method. The average service life was 20 years and the probable service lives for group properties were determined by using the S_3 or R_3 Iowa type survivor curves which are close to the survivor curve of the oil refineries. The results of the ratio method seem to be reliable since the method uses the exact operation returns data. Thus, the results of the ratio method would be good criteria for checking the results of the Delta method in which service intensity (reduction of revenue) and repair and maintenance costs (operating costs) were used to determine the close actual operation returns. The K values of the Delta

Table 7.5. K values of ratio method

Refinery	R_1	R_x	X	PL	Ratio (%)	K value
α-1	14.7	12.0	5	20	81.63	0.86
α-2	14.7	8.9	10	21	60.54	0.77
α-3	14.7	6.2	16	22	42.18	0.74
	23.2	11.0	10	21	47.41	0.55

method for the refineries were derived form the data of increasing operating costs and decreasing gross revenue. The procedure to get the Δ and K values is shown in Table 7.6.

The K values of a ratio method and Delta method were represented in Table 7.7 for comparison. The K values of ratio method and those of Delta method were close to each other. Therefore, the assumptions for the group property procedures as well as the unit property procedures are proved to be reasonable and realistic. These assumptions are that increasing repair and maintenance costs and decreasing service intensity with the passage of time are the main factors of declining operation returns for unit properties, and increasing operating costs and decreasing gross operating revenue are the main factors for group properties. Thus, the K values of a Alpha and Beta oil refineries were determined to be 0.75 and 0.54, respectively. The value of age x can be derived from the K values obtained.

The comparisons between V_x of unit property and group property

Table 7.6. K values of Delta method

Refinery	Age	Increasing of Operating Costs	Decreasing of Gross Revenue	Δ	Δ/V_N	K
α-1	5	1.0	2.9	3.9	0.026	0.83
α-2	10	3.6	5.1	8.7	0.058	0.76
α-3	16	5.7	7.5	13.2	0.088	0.74
	10	10.0	12.9	22.9	0.093	0.53

Table 7.7. Comparison of K values of ratio method with Delta method

Refinery	Age	K value of Ratio Method	K value of DELTA method
α-1	5	0.86	0.83
α-2	10	0.77	0.76
α-3	16	0.74	0.74
	10	0.55	0.53

were considered to estimate the differences between them. The graphical comparison of V_x are shown in Figures 7.5 and 7.6. There are three graphs, for example, that represent the V_x for each R_1, R_3, and R_5 Iowa type survivor curves for group property and one graph for R_5 Iowa type unit property. Consideration of L and S Iowa type survivor curves was skipped because of little influence of those to determine the V_x [14]. These figures are the results from K values of 0.6 and 0.9 with common conditions of 10 year probable service life, 10 percent salvage ratio and 7 percent discount rate. These figures show that the group basis with frequency of equal life group does not make a big difference with unit basis even though slight differences occur when the dispersion pattern is quite different. It, therefore, seems that frequency for a group property normally have little effect on the analysis of V_x.

E. Standard Tables of Value of R and M and Intensity

Standard tables of value were developed for easy application in the industry since all the procedures had to be done for every determination of value for the properties in question. Thus, the role of the standard table is to reduce the complexity and time for the final goal of finding V_x. If the basic data, R and M costs and intensity rate, were given the value of age x can be easily obtained by using these standard tables. These standard tables start from setting the standard curves for each of R and M costs and intensity rates. With the standard curves, which can generally represent the normal shape of R and M cost and intensity rate, all the procedures will be completed for final standard tables.

These standard curve equations were determined by the trial and error method to find the best fitting general equations on the basis of collected data for this study.

The standard curve equation for service intensity rate is was set as:

$$I = J^{(x-1)} \tag{7.1}$$

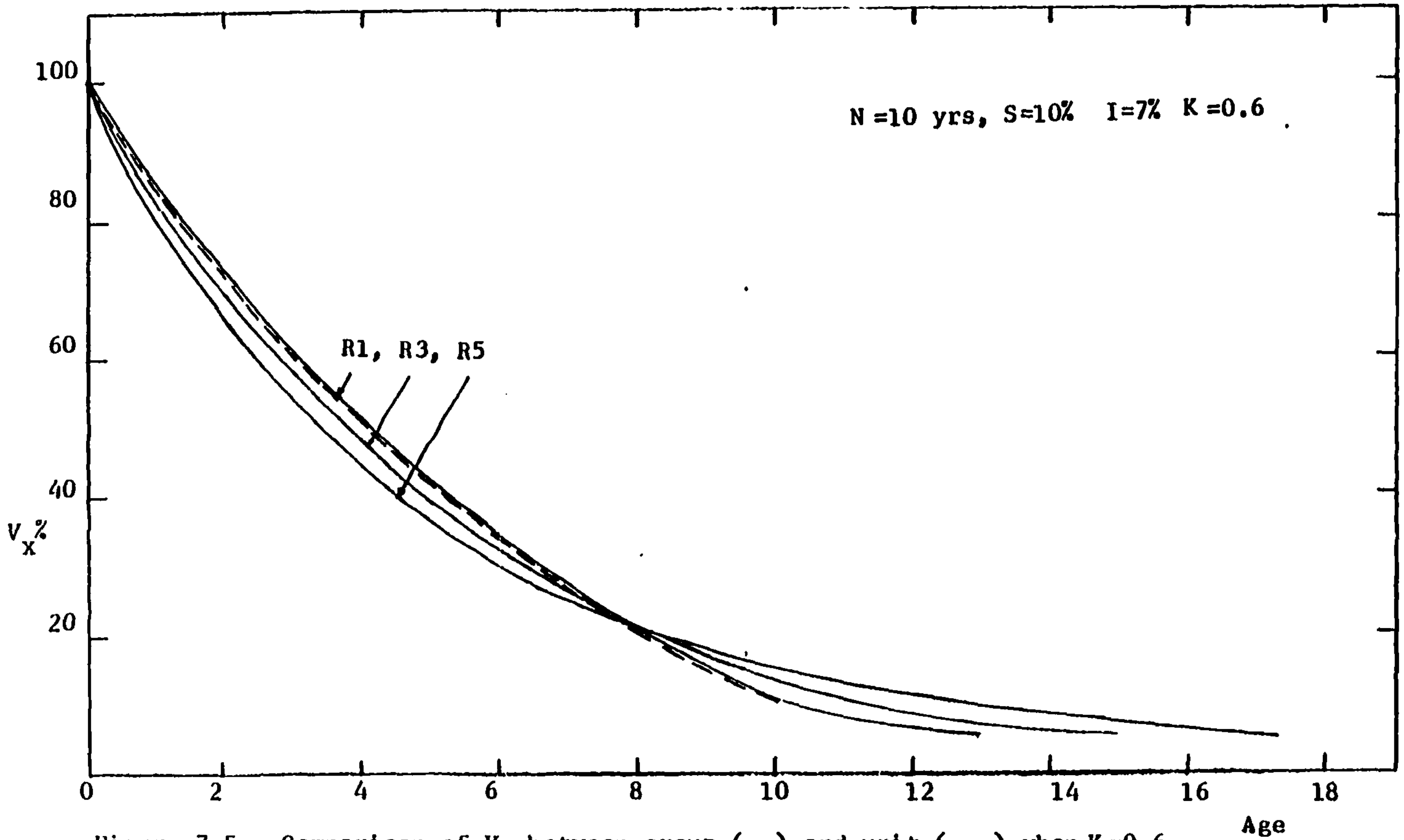

Figure 7.5. Comparison of V_x between group (—) and unit (---) when K =0.6

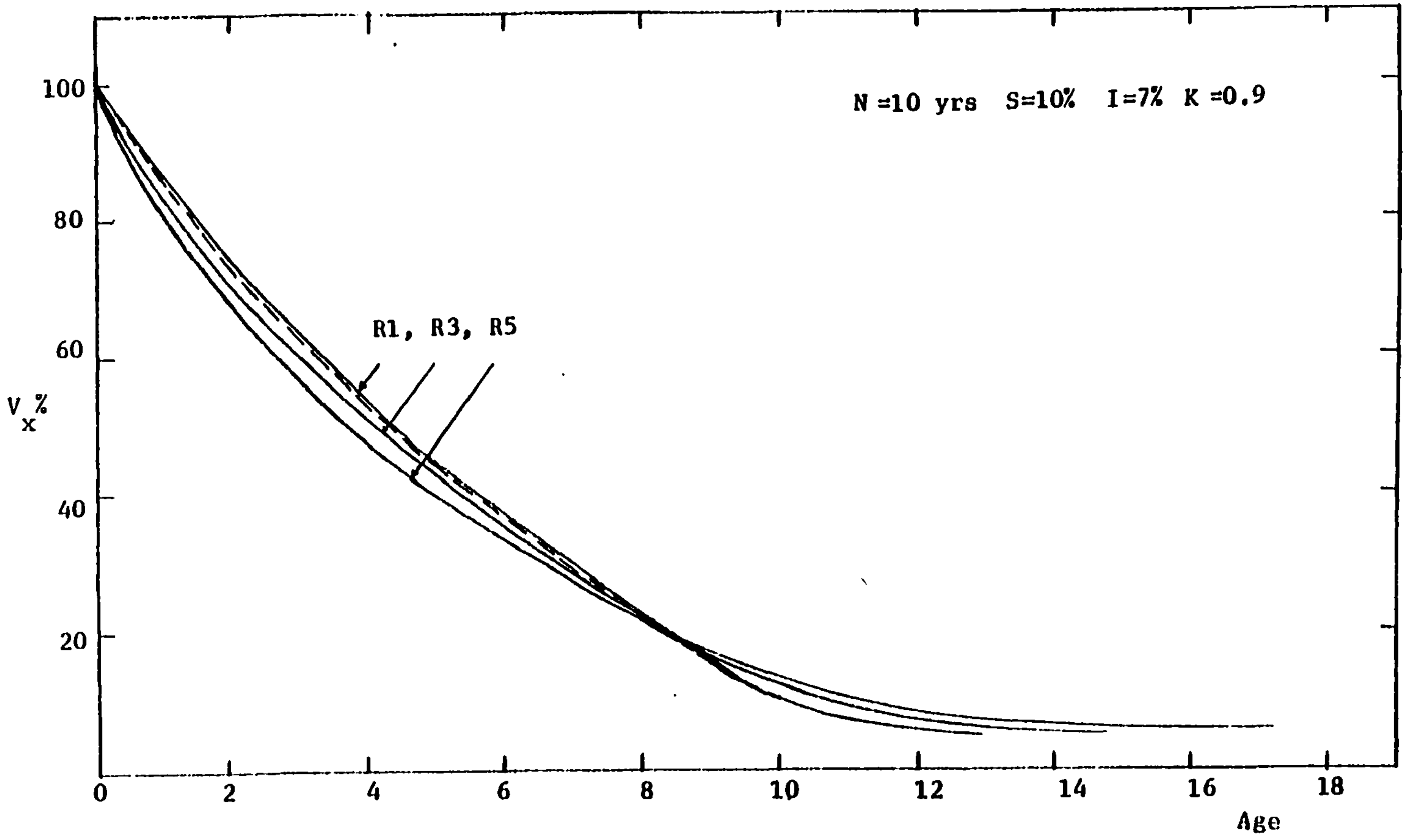

Figure 7.6. Comparison of V_x between group (—) and unit (---) when K =0.9

where I = service intensity rate,

x = age, and

J = variable.

The standard curve equation for R and M costs was determined as:

$$R = H(x-1)^{.67} \tag{7.2}$$

where R = R and M costs/V_N

H = variable.

Usually, the J factor of Equation (7.1) in service intensity rate equation changes from 0.86 to 1.0, and the H factor of Equation (7.2) varies 0 to 0.04 in normal situations. Thus, intensity curves were divided into four cases in the above range with the value of J as .98, .94, .90, and .86. R and M curves were also divided into four according to the value of H as .01, .02, .03, and .04. The standard curves of R and M costs begin at 0% of age 1 so that the curves represent the differences between R and M costs of age X and those of age 1.

These two sets of four curves can be named as I_1, I_2, I_3, and I_4 in case of intensity and R_1, R_2, R_3, and R_4 for R and M curves, respectively. The composition of the combination of 4 Is and 4 Rs will work through the same procedures described before. The variables of these equations are also probable life (N) and salvage ratio (S). These standard curves were illustrated in Figures 7.7 and 7.8. The results of K values after some procedures with the variation of N and S were shown in Table 7.5. As expected, the effect of H, J, and probable life (N) was significant but effect salvage rate (S) was quite small and negligible within the normal range.

The values with passage of age (V_x) corresponding to K value, probable life (N), and salvage ratio (S) in the standard table were listed in Appendix C. Thus, the applications of the theories of this thesis become very simple. The procedures to obtain the V_x value of a piece of equipment are just four steps as follows:

1. Prepare R and M costs data, service intensity data, probable life (N), and salvage ratio (S) of a property in question.
2. Match the standard curves with the R and M cost data and service intensity data and find out which curve fits best for each of data. For example, R_1 for R and M costs data and I_3 for intensity data.
3. Find the K value for obtained standard curves (R_1, I_3) in the standard table.
4. Find the V_x value corresponding to the K value obtained with a given probable life (N) and salvage ratio (S).

As a specific example, the data for 1.50 ton C. crane were used. The data from Table 6.6 and the smoothed curve of R and M costs from Table 6.7 were drawn in Figure 7.7 as points and a dotted line. The value at age 1 was 0.0204 which was derived from the smoothed curve for drawing the points. This illustration shows that the R and M costs curve is close to the R_2 standard curve. Also, the smoothed curve for the intensity rates from Table 6.9 is illustrated in Figure 7.8 as well as the points of original data from Table 6.8. This figure shows that the curve is close to the I_2 standard curve. The probable life of the equipment is 15 years and the salvage rate is 10 percent. These informations are then applied to Table 7.8 for the K value of it. The K value for these informations is found as 0.4. With this K value, the value at age X can be obtained from the standard table of value at Appendix C.

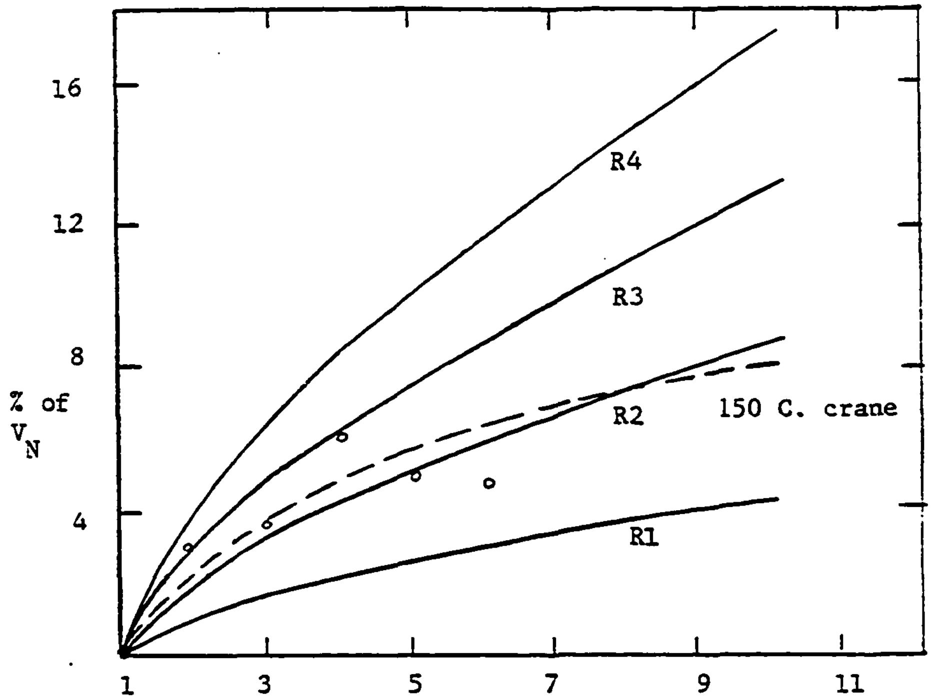

Figure 7.7. Standard curves for R and M costs

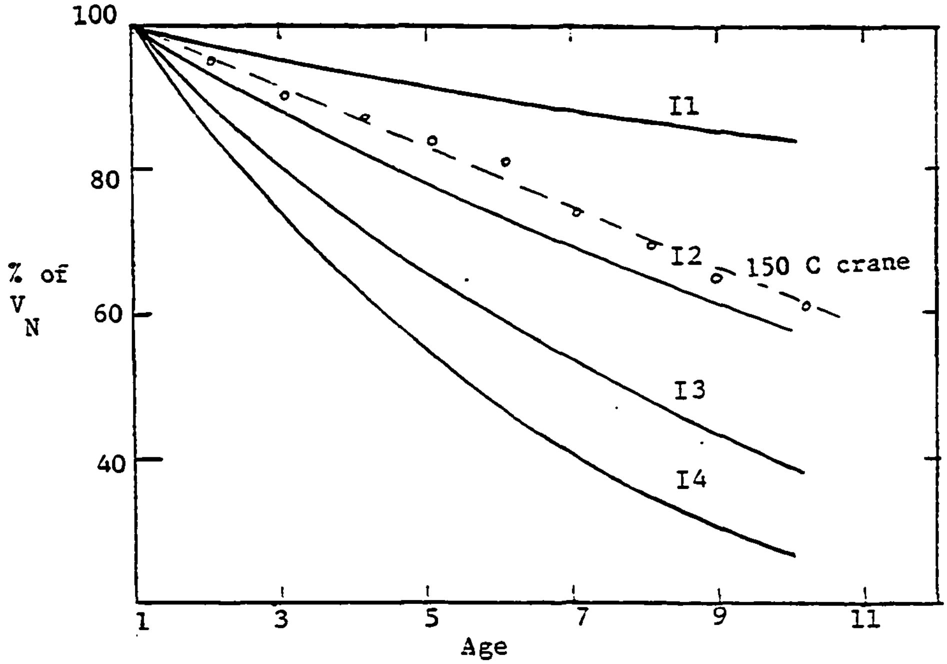

Figure 7.8. Standard curves for intensity rate

Table 7.8. K values with the variation of R and I

N (yr.)		10			15		
R	I S(%)	10	15	20	10	15	20
R_1	I_1	1.00	1.00	1.00	.85	.85	.85
	I_2	.82	.82	.82	.58	.58	.55
	I_3	.65	.62	.62	.40	.40	.39
	I_4	.50	.48	.48	.28	.28	.26
R_2	I_1	.85	.85	.85	.58	.58	.58
	I_2	.62	.62	.62	.40	.40	.40
	I_3	.50	.49	.48	.28	.28	.28
	I_4	.39	.38	.38	.20	.20.	.20
R_3	I_1	.65	.65	.65	.45	.45	.42
	I_2	.52	.52	.50	.30	.30	.30
	I_3	.40	.40	.40	.22	.22	.20
	I_4	.32	.30	.30	.12	.12	.12
R_4	I_1	.55	.55	.52	.32	.32	.32
	I_2	.42	.42	.42	.25	.25	.25
	I_3	.32	.32	.32	.15	.15	.14
	I_4	.25	.25	.25	.12	.12	.12

VIII. CONCLUSIONS AND FURTHER STUDY

A. Conclusions

Conclusions were drawn from this research as follows:

1. The proposed Y model appears to be valid. The Y model is simple to apply compared to the Elfar model. Furthermore, the value of the Y model fits close to the market evidence value for all of the data used. Thus, the Y model appears to be better than the Elfar model in its ability to predict value curves and application.
2. There are several factors that reduce the operation returns of an industrial property with the passage of time. Among these factors, two factors were chosen and analyzed. These are the increasing repair and maintenance costs and the decreasing service intensity rate for unit property. From the results, the proposed Delta method using R and M costs and service intensity rates appears to valid in fitting with market evidence value.
3. The two factors chosen for group property which are comparable to those of unit property are the increasing operating costs and decreasing gross operating revenue. The slope factor values of the Delta method using these two factors agree with the results of the ratio method which uses the operating returns directly. Therefore, the proposed Delta method for group property also seems valid in valuation procedures.
4. As a result of the above (No. 2 and 3), the R and M costs

(operating costs) and service intensity rates (operating revenue) appear to be the main factors of declining operation returns of industrial property. Furthermore, the service intensity rate seems to include other factors like obsolescence, downtime cost, decrease in production rate, etc.

5. The market evidence appears to have a tendency to rapidly drop in value in the early part of life and to slowly decline for the rest of its life for all the properties. In case of the intensity rate, the decline in the early part of life was usually bigger than that of the rest of life and these graphs were usually in a convex shape. In contrast, the shape of R and M costs graph was usually concave, showing that the increase in costs in the early part of life is bigger than the increase in the latter part of life.
6. If repair and maintenance costs data, average life and salvage ratio at retirement are provided, the value at age x which is reasonably close to the market evidence can be obtained using the procedure of this study for the industrial property.
7. The standard curves for R and M costs and intensity rate and the standard tables were developed. These standard curves and tables are used for easy application of the procedures of this dissertation to determine the value of age x.

B. Further Study

Further study is recommended in the following areas:

1. Collection of data for the actual change of operation returns over the passage of age for unit and group properties and comparison of these data with the models described in this dissertation.
2. Study of market evidence which decreases greatly in value in the early part of property life.
3. Finding the relationship between the intensity rates and the other factors, especially obsolescence, in the decrease of operation returns.
4. Developing the procedure and proving that the weighted average of slope factor values of each frequency group of a group property would be the slopoe factor value of the whole group property.

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XI. APPENDIX A. TABLES

Table 11.1. Repair and maintenance costs data

	Gas Tractor	Diesel Tractor		Combine
Cum. Hrs.	Cum. R&M Costs/V_N	Cum. R&M Costs/V_N	Cum. Acres	Cum. R&M Costs/V_N
400	0.001	0.001	600	0.003
800	0.011	0.007	1200	0.01
1200	0.032	0.016	1800	0.027
1600	0.064	0.033	2400	0.052
2000	0.1	0.05	3000	0.08
2400	0.14	0.071	3600	0.12
2800	0.184	0.092	4200	0.165
3200	0.228	0.113	4800	0.218
3600	0.274	0.134	5400	0.285

Table 11.1. (continued)

Corn Picker		Forage Harvester		Baler	
Cum. Hrs.	Cum. R&M Costs/V_N	Cum. Acres	Cum. R&M Costs/V_N	Cum. Acres	\$/Yr./\$1000 of V_N
100	0.0005	200	0.018	80	0.212
200	0.002	400	0.052	110	0.375
300	0.004	600	0.095	240	0.497
400	0.009	800	0.141	320	0.597
500	0.015	1000	0.193	400	0.650
600	0.025	1200	0.243	480	0.686
700	0.035	1400	0.302	560	0.716
800	0.048	1600	0.358	640	0.740
900	0.064				
1000	0.082				
1100	0.104				
1200	0.138				

Table 11.2. Repair and maintenance costs data

	35 ton end dump truck		50 ton end dump truck		D9 dozer	
Hours range	Repair hrs/ Machine hrs	Parts cost ($)/Machine hrs	R hrs/ M hrs	Parts cost/ M hrs	R hrs/ M hrs	Parts Cost/ M hrs
0 ~ 2000	0.038	18.76	0.104	3.664	0.088	8.478
2000 ~ 4000	0.040	4.402	0.110	1.830	0.154	10.264
4000 ~ 6000	0.093	4.342	0.170	5.597	0.205	18.833
8000 ~ 10000	0.189	20.0	0.335	21.451	0.298	33.066
10000 ~ 12000	0.111	4.308	0.351	26.817	0.310	17.036

Table 11.2. (continued)

	D9 C dozer		10 M gal. water tanker		Cat. 637 scraper	
Hours range	R hrs/ M hrs	Parts cost/ M hrs	R hrs/ M hrs	Parts cost/ M hrs	R hrs/ M hrs	Parts Cost/ M hrs
0 ~ 2000	0.042	11.076	0.109	7.864	.342	20.305
2000 ~ 4000	0.134	16.568	0.159	11.292	.334	18.897
4000 ~ 6000	0.214	22.495	0.209	14.585	.470	25.076
6000 ~ 8000	0.254	30.977	0.243	17.523	.654	31.746
8000 ~ 10000	0.281	39.394	0.260	18.801	.735	27.896
10000 ~ 12000	0.293	45.132	0.262	17.151	.507	24.784
12000 ~ 14000	0.294	45.548				

Table 11.4. Repair and maintenance costs data

	Cat. 666 Scraper		Cat. 825 Compactor		900 CFM Por. Air Compressor	
Hours range	R hrs/ M hrs	Parts cost/ M hrs	R hrs/ M hrs	Parts cost/ M hrs	R hrs/ M hrs	Parts Cost/ M hrs
0 ~ 2000	0.178	10.523	0.165	5.525	0.037	0.89
2000 ~ 4000	0.380	21.489	0.177	7.215	0.049	2.63
4000 ~ 6000	0.447	29.056	0.132	8.638	0.046	2.579
6000 ~ 8000	0.467	37.652	0.322	23.158	0.056	1.965
8000 ~ 10000	0.563	51.430	0.199	11.134	0.092	2.021
10000 ~ 12000	0.833	74.538	0.199	11.134	0.164	7.989

Table 11.3. (continued)

	150 Ton Crawler Crane	
Hours range	R hrs/ M hrs	Parts cost/ M hrs
0 ~ 2000	0.18	19.973
2000 ~ 4000	0.222	14.389
4000 ~ 6000	0.234	24.312
6000 ~ 8000	0.212	19.558
8000 ~ 10000	0.212	19.558
10000 ~ 12000		

Table 11.4. Repair and maintenance costs data

	6-in. Rotary Drill	
Hours Range	Repair Hrs./Machine Hrs.	Parts Cost ($)/Machine Hrs.
0 - 1000	0.189	3.64
1000 - 2000	0.229	3.82
2000 - 3000	0.233	2.60
3000 - 4000	0.256	3.37
4000 - 5000	0.289	5.47
5000 - 6000	0.324	8.22
6000 - 7000	0.352	10.96
7000 - 8000	0.367	13.01
8000 - 9000	0.358	13.69
9000 - 10000	0.318	12.33

Table 11.4. (continued)

35 ton Motor Crane

Hours Range	Repair Hrs./Machine Hrs.	Parts Cost ($)/Machine Hrs.
0 - 1000	0.072	5.02
1000 - 2000	0.046	0.53
2000 - 3000	0.059	0.15
3000 - 4000	0.116	3.56
4000 - 5000	0.178	8.13
5000 - 6000	0.207	11.24
6000 - 8000	0.145	7.91
8000 - 10000	0.121	5.554
10000 - 12000	0.121	5.554

80-KW DSL Generator

Hours Range	Repair Hrs./Machine Hrs.	Parts Cost ($)/Machine Hrs.
0 - 1000	0.0113	0.1914
1000 - 2000	0.0065	0.75
2000 - 3000	0.0115	1.314
3000 - 4000	0.0152	2.253
4000 - 6000	0.012	1.127
6000 - 8000	0.012	1.127
8000 - 10000	0.012	1.127

Table 11.5. Repair and maintenance costs data ($)

Age	D8 Dozer	Industrial Forklift	Pickup Truck
1	20750	385	942
2	21680	590	1250
3	25130	660	1328
4	26720	1205	920
5	26880	1325	738
6	31960	1070	
7	34384	1385	
8		1705	
9		1560	

XII. APPENDIX B. STANDARD TABLE OF DELTA METHOD

PROBABLE LIFE 10 SALVAGE RATIO .1

K VALUES ARE	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2
DELTA % AT AGE 1	0	0	0	0	0	0	0	0	0	0	0
DELTA % AT AGE 2	23.91033	14.8547	9.798362	6.706623	4.705643	3.360143	2.430605	1.775568	1.307006	.9679378	.7203361
DELTA % AT AGE 3	27.46576	18.28828	12.92902	9.484598	7.13242	5.458568	4.23193	3.313327	2.614012	2.07482	1.654698
DELTA % AT AGE 4	29.78584	20.65381	15.20555	11.61621	9.096862	7.250077	5.853444	4.772536	3.921019	3.241014	2.692036
DELTA % AT AGE 5	31.54987	22.51551	17.05994	13.41324	10.81073	8.867471	7.368218	6.182887	5.228025	4.447475	3.801959
DELTA % AT AGE 6	32.98979	24.07436	18.65269	14.99646	12.3595	10.36661	8.808266	7.558061	6.535031	5.684792	4.969347
DELTA % AT AGE 7	34.21494	25.42783	20.06384	16.4278	13.78827	11.77779	10.19144	8.905811	7.842038	6.947266	6.184675
DELTA % AT AGE 8	35.28623	26.63135	21.33992	17.74406	15.12439	13.11978	11.52903	10.23118	9.149044	8.231053	7.441373
DELTA % AT AGE 9	36.24128	27.71985	22.51072	18.95919	16.386	14.40525	12.82881	11.53768	10.45605	9.533371	8.734606
DELTA % AT AGE 10	37.10514	28.71653	23.59566	20.11957	17.58539	15.64327	14.09641	12.8279	11.76306	10.85211	10.06066

```
PROBABLE LIFE  10          SALVAGE RATIO  .15

         K VALUES ARE      .2            .3            .4            .5

DELTA % AT AGE 1           0             0             0             0
DELTA % AT AGE 2           23.27005      14.45691      9.535974      6.527028
DELTA % AT AGE 3           26.73026      17.79854      12.58279      9.230612
DELTA % AT AGE 4           28.98821      20.10073      14.79836      11.30514
DELTA % AT AGE 5           30.705        21.91257      16.6031       13.05405
DELTA % AT AGE 6           32.10637      23.42968      18.1532       14.59488
DELTA % AT AGE 7           33.29871      24.7469       19.52655      15.98789
DELTA % AT AGE 8           34.3413       25.9182       20.76846      17.26889
DELTA % AT AGE 9           35.27079      26.97754      21.90792      18.46122
DELTA % AT AGE 10          36.11151      27.94784      22.96477      19.58108



                           .6            .7            .8            .9

                           0             0             0             0
                           4.579632      3.270163      2.365516      1.72802
                           6.941423      5.312394      4.118604      3.224601
                           8.85326       7.055929      5.676696      4.544704
                           10.53123      8.630011      7.170906      6.017318
                           12.02853      10.08901      8.572391      7.355665
                           13.41903      11.46239      9.918523      8.667324
                           14.71938      12.76844      11.2203       9.957198
                           15.9472       14.01949      12.48527      11.22871
                           17.11497      15.22436      13.71892      12.48439



                           1             1.1           1.2

                           0             0             0
                           1.272006      .9420177      .7010465
                           2.544012      2.019259      1.610582
                           3.816019      3.154224      2.619947
                           5.088025      4.328377      3.700146
                           6.360032      5.532561      4.836274
                           7.632037      6.761228      6.019057
                           8.904044      8.010636      7.242102
                           10.17605      9.27808       8.500704
                           11.44806      10.56151      9.791246
```

PROBABLE LIFE 15 SALVAGE RATIO .1

K VALUES ARE	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2
DELTA % AT AGE 1	0	0	0	0	0	0	0	0	0	0	0
DELTA % AT AGE 2	17.80645	10.40439	6.522973	4.264303	2.864936	1.961644	1.361764	.9551328	.6752601	.480379	.3434458
DELTA % AT AGE 3	20.45424	12.80931	8.607114	6.030635	4.342431	3.1867	2.37097	1.782341	1.350521	1.029715	.7890312
DELTA % AT AGE 4	22.18205	14.46615	10.12265	7.38599	5.538443	4.232579	3.279436	2.567279	2.025731	1.608487	1.283524
DELTA % AT AGE 5	23.49576	15.77011	11.35716	8.528606	6.581894	5.176811	4.128099	3.325966	2.701042	2.207243	1.812719
DELTA % AT AGE 6	24.56809	16.86194	12.41749	9.535271	7.524835	6.052006	4.934898	4.065714	3.376303	2.821313	2.369313
DELTA % AT AGE 7	25.48048	17.80992	13.35692	10.44537	8.394711	6.875847	5.70983	4.790712	4.051563	3.447868	2.948763
DELTA % AT AGE 8	26.27829	18.65289	14.20643	11.28229	9.208179	7.659298	6.459227	5.503667	4.726825	4.085	3.547939
DELTA % AT AGE 9	26.98953	19.41528	14.98586	12.06127	9.976286	8.409753	7.187438	6.206473	5.402084	4.731329	4.164534
DELTA % AT AGE 10	27.63287	20.11358	15.70879	12.79291	10.70682	9.132509	7.897619	6.900525	6.077345	5.385911	4.796777
DELTA % AT AGE 11	28.22132	20.75949	16.38497	13.48491	11.40552	9.831511	8.592156	7.586892	6.752606	6.047617	5.443253
DELTA % AT AGE 12	28.76444	21.36163	17.02169	14.14309	12.07676	10.50982	9.272914	8.266416	7.427865	6.716085	6.102807
DELTA % AT AGE 13	29.26939	21.92659	17.62455	14.77198	12.724	11.16985	9.94139	8.939784	8.103126	7.390666	6.77448
DELTA % AT AGE 14	29.74172	22.45948	18.19797	15.37516	13.34999	11.81356	10.5988	9.607556	8.778386	8.0709	7.457453
DELTA % AT AGE 15	30.18582	22.9644	18.74549	15.95556	13.95699	12.44257	11.24617	10.27021	9.453649	8.756391	8.151024

PROBABLE LIFE 15 SALVAGE RATIO .15

K VALUES ARE	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2
DELTA % AT AGE 1	0	0	0	0	0	0	0	0	0	0	0
DELTA % AT AGE 2	17.47162	10.20875	6.400316	4.184117	2.811064	1.924759	1.336158	.9371728	.6625626	.471346	.3369877
DELTA % AT AGE 3	20.06962	12.56844	8.445268	5.917237	4.260777	3.126778	2.326337	1.748626	1.325126	1.010353	.7741945
DELTA % AT AGE 4	21.76494	14.19413	9.932303	7.247106	5.4343	4.152991	3.21777	2.519004	1.987689	1.578241	1.259389
DELTA % AT AGE 5	23.05395	15.47357	11.1436	8.368235	6.45813	5.079466	4.050475	3.263425	2.650252	2.165738	1.778633
DELTA % AT AGE 6	24.10612	16.54488	12.18399	9.355972	7.383339	5.938205	4.842103	3.989263	3.312816	2.768262	2.324761
DELTA % AT AGE 7	25.00135	17.47503	13.10576	10.24895	8.226658	6.746555	5.602463	4.700629	3.975378	3.383035	2.893315
DELTA % AT AGE 8	25.78416	18.30214	13.93929	11.07014	9.035031	7.515273	6.33777	5.400178	4.637942	4.008187	3.481224
DELTA % AT AGE 9	26.48203	19.0502	14.70407	11.83447	9.788696	8.251618	7.052287	6.089768	5.300505	4.642352	4.086225
DELTA % AT AGE 10	27.11326	19.73537	15.4134	12.55235	10.50549	8.960753	7.749114	6.770769	5.963068	5.284537	4.706579
DELTA % AT AGE 11	27.69066	20.36913	16.07687	13.23134	11.19105	9.646642	8.430591	7.44423	6.625631	5.932999	5.340899
DELTA % AT AGE 12	28.22356	20.95995	16.70162	13.87715	11.84967	10.31219	9.098549	8.110977	7.288193	6.589797	5.988052
DELTA % AT AGE 13	28.71901	21.51428	17.29314	14.49421	12.48474	10.95981	9.754454	8.771682	7.950757	7.251694	6.647094
DELTA % AT AGE 14	29.19246	22.03715	17.85578	15.08605	13.09876	11.59142	10.3995	9.426898	8.613319	7.919136	7.317224
DELTA % AT AGE 15	29.61822	22.53259	18.39301	15.65554	13.69455	12.2086	11.0347	10.07709	9.275884	8.591738	7.997754

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PROBABLE LIFE  20           SALVAGE RATIO  .1

             K VALUES ARE   .2          .3          .4          .5          .6

DELTA % AT AGE 1            0           0           0           0           0
DELTA % AT AGE 2            14.53909    8.173375    4.959247    3.145608    2.05301
DELTA % AT AGE 3            16.70102    10.06261    6.543766    4.448562    3.111782
DELTA % AT AGE 4            18.11179    11.36417    7.695986    5.448354    3.968842
DELTA % AT AGE 5            19.18444    12.38852    8.634551    6.291217    4.716579
DELTA % AT AGE 6            20.06001    13.24624    9.44069     7.033794    5.392288
DELTA % AT AGE 7            20.80498    13.99094    10.15491    7.705135    6.01564
DELTA % AT AGE 8            21.45639    14.65315    10.80078    8.322496    6.598571
DELTA % AT AGE 9            22.03713    15.25206    11.39336    8.897123    7.148996
DELTA % AT AGE 10           22.56242    15.80062    11.94298    9.436824    7.672494
DELTA % AT AGE 11           23.0429     16.30803    12.45707    9.947287    8.17318
DELTA % AT AGE 12           23.48636    16.78106    12.94115    10.4328     8.654194
DELTA % AT AGE 13           23.89865    17.22487    13.39949    10.89671    9.118004
DELTA % AT AGE 14           24.28431    17.64349    13.83545    11.34165    9.566588
DELTA % AT AGE 15           24.64692    18.04014    14.25171    11.76979    10.00156
DELTA % AT AGE 16           24.98937    18.41742    14.65049    12.18289    10.42427
DELTA % AT AGE 17           25.31402    18.77749    15.03363    12.58243    10.83595
DELTA % AT AGE 18           25.62282    19.12212    15.40265    12.96967    11.23726
DELTA % AT AGE 19           25.91741    19.45285    15.75886    13.34558    11.62933
DELTA % AT AGE 20           26.19918    19.77095    16.10339    13.71139    12.01277

                .7          .8          .9          1           1.1         1.2

                0           0           0           0           0           0
                1.366433    .9222656    .6291654    .4326187    .2993333    .2081413
                2.219779    1.605932    1.17407     .8652377    .6416351    .4781834
                2.948312    2.221264    1.691127    1.297857    1.002279    .7778645
                3.606041    2.796091    2.190893    1.730475    1.375375    1.098577
                4.21568     3.34256     2.678182    2.163395    1.758013    1.435895
                4.789547    3.867447    3.155754    2.595713    2.148432    1.787064
                5.33528     4.373036    3.625394    3.028332    2.545441    2.150187
                5.858031    4.968277    4.08835     3.460951    2.948181    2.522867
                6.361482    5.349304    4.545539    3.89357     3.255999    2.90703
                6.848393    5.819736    4.997665    4.326189    3.768385    3.29882
                7.320884    6.280836    5.445283    4.758807    4.184919    3.698535
                7.780644    6.733615    5.888846    5.191427    4.605265    4.105595
                8.229036    7.1789      6.328723    5.624045    5.02913     4.519503
                8.667189    7.617381    6.765226    6.056665    5.456273    4.939633
                9.096042    8.049636    7.198619    6.489283    5.886478    5.366215
                9.516398    8.476161    7.62913     6.921902    6.319565    5.798325
                9.928939    8.897386    8.056956    7.354521    6.755368    6.235873
                10.33426    9.31368     8.482274    7.78714     7.193743    6.678603
                10.73288    9.725371    8.905233    8.219759    7.634563    7.126281
```

PROBABLE LIFE 20 SALVAGE RATIO .15

K VALUES ARE	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2
DELTA % AT AGE 1	0	0	0	0	0	0	0	0	0	0	0
DELTA % AT AGE 2	14.34624	8.064966	4.893469	3.103886	2.025779	1.34831	.9101316	.6208232	.4268806	.295363	.2053806
DELTA % AT AGE 3	16.4795	9.929139	6.456971	4.389558	3.070508	2.190336	1.584632	1.158497	.8537614	.6331246	.4718408
DELTA % AT AGE 4	17.87156	11.21344	7.593909	5.376088	3.9162	2.909207	2.191602	1.666696	1.220642	.9889848	.7675472
DELTA % AT AGE 5	18.92998	12.2242	8.520024	6.207772	4.654019	3.558211	2.759004	2.161833	1.707523	1.357132	1.084006
DELTA % AT AGE 6	19.79394	13.07054	9.315472	6.940499	5.320766	4.159764	3.298225	2.642659	2.134404	1.734695	1.416849
DELTA % AT AGE 7	20.52903	13.80537	10.02022	7.602936	5.93585	4.726021	3.81615	3.112997	2.561284	2.119935	1.763361
DELTA % AT AGE 8	21.1718	14.45879	10.65752	8.212109	6.511049	5.264515	4.317007	3.577308	2.988165	2.511679	2.121667
DELTA % AT AGE 9	21.74484	15.04976	11.24224	8.779115	7.054174	5.780331	4.803705	4.034123	3.415046	2.909078	2.490391
DELTA % AT AGE 10	22.26315	15.59105	11.78457	9.311657	7.570728	6.277106	5.278353	4.465247	3.841927	3.311486	2.868472
DELTA % AT AGE 11	22.73726	16.09173	12.29184	9.815348	8.064772	6.757553	5.742544	4.931377	4.268808	3.718402	3.255065
DELTA % AT AGE 12	23.17484	16.55648	12.7695	10.29442	8.539408	7.223781	6.197528	5.373059	4.695688	4.129412	3.649479
DELTA % AT AGE 13	23.58166	16.9964	13.22176	10.75219	8.997066	7.677444	6.644302	5.810738	5.122569	4.544182	4.051139
DELTA % AT AGE 14	23.96221	17.40947	13.65194	11.19122	9.439699	8.119888	7.083681	6.24478	5.54945	4.962425	4.459557
DELTA % AT AGE 15	24.32001	17.80086	14.06266	11.61368	9.868935	8.552229	7.516347	6.675493	5.976331	5.383902	4.874312
DELTA % AT AGE 16	24.65792	18.17314	14.45617	12.0213	10.28601	8.975394	7.942868	7.103139	6.403211	5.808402	5.295039
DELTA % AT AGE 17	24.97626	18.52843	14.83422	12.41554	10.69213	9.390175	8.363735	7.52794	6.830092	6.235744	5.721418
DELTA % AT AGE 18	25.28297	18.86849	15.19835	12.79765	11.08821	9.797244	8.779372	7.950092	7.256972	6.665766	6.153163
DELTA % AT AGE 19	25.57365	19.19483	15.54984	13.16667	11.47508	10.19719	9.190144	8.369768	7.683854	7.098328	6.590019
DELTA % AT AGE 20	25.85169	19.50871	15.88979	13.52952	11.85344	10.59052	9.596376	8.787117	8.110733	7.5333	7.03176

XIII. APPENDIX C. STANDARD TABLE OF VALUES WITH VARIATION OF K, PL, AND S

PROBABLE LIFE 10 SALVAGE RATIO .1

K VALUES ARE	.2	.3	.4	.5	.6	.7
% OF VN AT AGE 0	100	100	100	100	100	100
% OF VN AT AGE 1	66.30268	74.01366	78.61335	81.65135	83.79616	85.38332
% OF VN AT AGE 2	54.15687	61.06295	65.528	68.72491	71.16368	73.10361
% OF VN AT AGE 3	44.71628	50.63928	54.65732	57.67161	60.07371	62.06276
% OF VN AT AGE 4	36.93492	41.85147	45.30222	47.9762	50.17189	52.04055
% OF VN AT AGE 5	30.3729	34.31024	37.14666	39.39913	41.29079	42.93417
% OF VN AT AGE 6	24.79147	27.79996	30.01297	31.80489	33.3368	34.68949
% OF VN AT AGE 7	20.04448	22.18742	23.79105	25.11038	26.25479	27.27886
% OF VN AT AGE 8	16.03648	17.38553	18.40968	19.26352	20.01316	20.69147
% OF VN AT AGE 9	12.70299	13.336	13.82243	14.23251	14.59622	14.92843
% OF VN AT AGE 10	10	10	10	10	10	10

K VALUES ARE	.8	.9	1	1.1	1.2
% OF VN AT AGE 0	100	100	100	100	100
% OF VN AT AGE 1	86.59972	87.55763	88.32849	88.95987	89.48466
% OF VN AT AGE 2	74.69205	76.01988	77.14698	78.11488	78.95365
% OF VN AT AGE 3	63.75215	65.21223	66.48977	67.61761	68.62001
% OF VN AT AGE 4	53.66798	55.10723	56.39355	57.55174	58.60015
% OF VN AT AGE 5	44.39268	45.70527	46.89762	47.98771	48.98881
% OF VN AT AGE 6	35.90816	37.02033	38.04396	38.9915	39.87207
% OF VN AT AGE 7	28.2129	29.07521	29.87756	30.62804	31.33248
% OF VN AT AGE 8	21.31656	21.89928	22.44651	22.96293	23.45183
% OF VN AT AGE 9	15.23726	15.52754	15.8023	16.06357	16.31275
% OF VN AT AGE 10	10	10	10	10	10

PROBABLE LIFE 10 SALVAGE RATIO .15

K VALUES ARE	.2	.3	.4	.5	.6	.7
% OF VN AT AGE 0	100	100	100	100	100	100
% OF VN AT AGE 1	67.3925	74.89699	79.37351	82.33014	84.41752	85.96219
% OF VN AT AGE 2	55.77252	62.49365	66.83915	69.95044	72.3239	74.2119
% OF VN AT AGE 3	46.79934	52.56373	56.47418	59.40776	61.74552	63.68131
% OF VN AT AGE 4	39.45599	44.2409	47.59923	50.2016	52.33849	54.15711
% OF VN AT AGE 5	33.31542	37.14731	39.90778	42.09993	43.94094	45.54031
% OF VN AT AGE 6	28.14635	31.07428	33.22802	34.97196	36.46285	37.77932
% OF VN AT AGE 7	23.8078	25.89335	27.45404	28.73804	29.8518	30.84845
% OF VN AT AGE 8	20.20814	21.52106	22.51779	23.34876	24.07832	24.73846
% OF VN AT AGE 9	17.28598	17.90205	18.37545	18.77454	19.12852	19.45183
% OF VN AT AGE 10	15	15	15	15	15	15

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                                 .8          .9          1          1.1         1.2

                            100         100         100         100         100
                            87.14602    88.07827    88.82849    89.44296    89.95371
                            75.75779    77.05006    78.14698    79.08896    79.90527
                            65.32546    66.74644    67.98977    69.08742    70.06296
                            55.74096    57.14168    58.39355    59.52072    60.54105
                            46.95977    48.2372     49.39762    50.45851    51.43281
                            38.96536    40.04774    41.04396    41.96613    42.82312
                            31.75747    32.59669    33.37756    34.10794    34.79352
                            25.34682    25.91393    26.44651    26.9491     27.42491
                            19.75239    20.0349     20.3023     20.55657    20.79908
                            15          15          15          15          15




PROBABLE LIFE  15          SALVAGE RATIO  .1

          K VALUES ARE       .2         .3         .4         .5         .6         .7

% OF VN AT AGE 0            100        100        100        100        100        100
% OF VN AT AGE 1            74.13179   80.9076    84.77495   87.26012   88.98182   90.23839
% OF VN AT AGE 2            64.25927   70.88308   75.00712   77.89275   80.05732   81.75514
% OF VN AT AGE 3            56.34345   62.56177   66.63967   69.636     71.98559   73.9031
% OF VN AT AGE 4            49.60135   55.31483   59.20205   62.15662   64.54486   66.5473
% OF VN AT AGE 5            43.70099   48.86453   52.4783    55.29631   57.62671   59.62083
% OF VN AT AGE 6            38.45994   43.05456   46.34421   48.96244   51.16725   53.08468
% OF VN AT AGE 7            33.76441   37.76587   40.72017   43.09529   45.12548   46.91486
% OF VN AT AGE 8            29.538     32.99133   35.55195   37.65436   39.47427   41.0966
% OF VN AT AGE 9            25.72699   28.62358   30.80139   32.61155   34.19552   35.62151
% OF VN AT AGE 10           22.29254   24.64837   26.44122   27.94738   29.27791   30.48593
% OF VN AT AGE 11           19.20613   21.04081   22.45202   23.64872   24.71471   25.68985
% OF VN AT AGE 12           16.44679   17.78287   18.82029   19.70734   20.50332   21.22636
% OF VN AT AGE 13           13.99924   14.86182   15.53721   16.11894   16.64438   17.13114
% OF VN AT AGE 14           11.8527    12.26919   12.59773   12.88255   13.1413    13.38228
% OF VN AT AGE 15           10         10         10         10         10         10
```

.8	.9	1	1.1	1.2
100	100	100	100	100
91.1913	91.9354	92.53014	93.01454	93.41531
83.12775	84.2614	85.21264	86.02047	86.71311
75.50895	76.87745	78.05818	79.08616	79.98735
68.26528	69.76154	71.07817	72.24523	73.2853
61.36323	62.90621	64.28483	65.52419	66.64329
54.78484	56.31076	57.69121	58.94673	60.09292
48.5209	49.97863	51.31129	52.53541	53.66348
42.56787	43.91619	45.16003	46.31242	47.38315
36.92634	38.13218	39.25345	40.30016	41.2798
31.60008	32.63736	33.60867	34.52152	35.38146
26.59553	27.44426	28.24402	29.00019	29.7167
21.92141	22.56717	23.1791	23.76083	24.31497
17.58858	18.02205	18.43489	18.82929	19.20679
13.60986	13.82654	14.03385	14.22278	14.42401
10	10	10	10	10

PROBABLE LIFE 15 SALVAGE RATIO .15

K VALUES ARE	.2	.3	.4	.5	.6	.7	.8	.9	1	1.1	1.2
% OF VN AT AGE 0	100	100	100	100	100	100	100	100	100	100	100
% OF VN AT AGE 1	74.74984	81.39823	85.19286	87.6313	89.32063	90.55358	91.48858	92.21868	92.80222	93.27752	93.67074
% OF VN AT AGE 2	65.20379	71.70306	75.74955	78.58092	80.70479	82.37068	83.71743	84.82981	85.76315	86.55581	87.22542
% OF VN AT AGE 3	57.58753	63.68892	67.69015	70.63012	72.93553	74.81699	76.39263	77.73541	78.89395	79.9026	80.73684
% OF VN AT AGE 4	51.13345	56.73949	60.55363	63.45263	65.79595	67.76076	69.44643	70.91455	72.20643	73.35154	74.37206
% OF VN AT AGE 5	45.51658	50.58303	54.12885	56.89386	59.18045	61.13706	62.84671	64.36066	65.71336	66.92941	68.02748
% OF VN AT AGE 6	40.55869	45.06692	48.29471	50.8637	53.02705	54.90844	56.57662	58.07395	59.42834	60.66026	61.79469
% OF VN AT AGE 7	36.149	40.09484	42.97396	45.30442	47.29643	49.05216	50.628	52.05832	53.36592	54.56702	55.67338
% OF VN AT AGE 8	32.21342	35.60182	38.11429	40.17716	41.96285	43.55467	44.99828	46.32125	47.5417	48.67242	49.72302
% OF VN AT AGE 9	28.70023	31.54235	33.67921	35.45533	37.00958	38.4087	39.68899	40.87216	41.97235	42.99938	43.96059
% OF VN AT AGE 10	25.57235	27.88389	29.64302	31.12086	32.42637	33.61167	34.70488	35.72265	36.6757	37.57139	38.41515
% OF VN AT AGE 11	22.80291	24.6031	25.98776	27.16196	28.20791	29.16471	30.05336	30.88613	31.67085	32.41281	33.11595
% OF VN AT AGE 12	20.37251	21.68347	22.70138	23.57175	24.35277	25.07202	25.74419	26.37781	26.97822	27.54901	28.09274
% OF VN AT AGE 13	18.26743	19.1138	19.77648	20.34728	20.86283	21.34045	21.78928	22.2146	22.61968	23.00666	23.37706
% OF VN AT AGE 14	16.47846	16.88712	17.20947	17.48894	17.74282	17.97927	18.20258	18.41519	18.61859	18.81378	19.00141
% OF VN AT AGE 15	15	15	15	15	15	15	15	15	15	15	15

PROBABLE LIFE 20 SALVAGE RATIO .1

K VALUES ARE	.2	.3	.4	.5	.6
% OF VN AT AGE 0	100	100	100	100	100
% OF VN AT AGE 1	78.57352	84.63497	88.04215	90.18602	91.6556
% OF VN AT AGE 2	70.18625	76.40916	80.20655	82.83068	84.78011
% OF VN AT AGE 3	63.37379	69.47536	73.40695	76.26342	78.4821
% OF VN AT AGE 4	57.49526	63.35779	67.2836	70.23622	72.60026
% OF VN AT AGE 5	52.27786	57.83634	61.67017	64.63	67.05447
% OF VN AT AGE 6	47.57082	52.78607	56.46995	59.37392	61.79616
% OF VN AT AGE 7	43.27925	48.12701	51.61994	54.42126	56.79313
% OF VN AT AGE 8	39.3387	43.80402	47.07629	49.73925	52.02281
% OF VN AT AGE 9	35.70304	39.77733	42.80717	45.30414	47.469
% OF VN AT AGE 10	32.33816	36.01733	38.78883	41.09828	43.11991
% OF VN AT AGE 11	29.21822	32.50153	35.00329	37.10847	38.96707
% OF VN AT AGE 12	26.32336	29.21267	31.43684	33.32488	35.00456
% OF VN AT AGE 13	23.63814	26.13738	28.07909	29.74035	31.22847
% OF VN AT AGE 14	21.15061	23.26545	24.92225	26.34984	27.63664
% OF VN AT AGE 15	18.85157	20.58914	21.96069	23.15014	24.22836
% OF VN AT AGE 16	16.73405	18.10277	19.19061	20.13956	21.00421
% OF VN AT AGE 17	14.79254	15.80241	16.60975	17.31778	17.96595
% OF VN AT AGE 18	13.02477	13.68567	14.21725	14.68571	15.11641
% OF VN AT AGE 19	11.4274	11.75148	12.0135	12.24542	12.45948
% OF VN AT AGE 20	10	10	10	10	10

.7	.8	.9	1	1.1	1.2
100	100	100	100	100	100
92.71961	93.52129	94.14399	94.63946	95.04141	95.37274
86.296	87.51141	88.50726	89.33631	90.03504	90.62975
80.27606	81.76443	83.02085	84.09456	85.02054	85.82478
74.5633	76.23048	77.66745	78.9185	80.01567	80.98313
69.10835	70.88399	72.43905	73.81274	75.03354	76.12327
63.8812	65.7097	67.33198	68.78219	70.08531	71.26055
58.86201	60.69812	62.34497	63.83212	65.18112	66.40861
54.03721	55.8433	57.47852	58.96817	60.33063	61.58015
49.39743	51.14189	52.73437	54.19637	55.54336	56.78739
44.93632	46.59241	48.11532	49.52313	50.82881	52.0423
40.64983	42.19488	43.62506	44.95541	46.1966	47.35683
36.53579	37.95064	39.2681	40.50055	41.65669	42.74309
32.59353	33.86207	35.04972	36.16648	37.21933	38.21347
28.82369	29.93259	30.97592	31.96164	32.89521	33.78066
25.22811	26.16653	27.05346	27.89508	28.69555	29.45789
21.80971	22.5691	23.28983	23.97648	24.63212	25.25891
18.57237	19.14637	19.69325	20.21619	20.71732	21.19812
15.52095	15.90528	16.27273	16.62531	16.96431	17.29061
12.66126	12.8536	13.0381	13.21568	13.38695	13.55231
10	10	10	10	10	10

PROBABLE LIFE 20 SALVAGE RATIO .15

K VALUES ARE	.2	.3	.4	.5	.6
% OF VN AT AGE 0	100	100	100	100	100
% OF VN AT AGE 1	78.93055	84.95136	88.2736	90.40904	91.85911
% OF VN AT AGE 2	70.77388	76.91426	80.66127	83.2506	85.17417
% OF VN AT AGE 3	64.15809	70.17873	74.05817	76.87675	79.066
% OF VN AT AGE 4	58.47125	64.25603	68.12977	71.04323	73.37593
% OF VN AT AGE 5	53.44476	58.92952	62.7125	65.63308	68.02538
% OF VN AT AGE 6	48.93038	54.07646	57.71148	60.57692	62.96704
% OF VN AT AGE 7	44.83507	49.61854	53.06513	55.82929	58.1697
% OF VN AT AGE 8	41.09587	45.50197	48.73083	51.35848	53.61175
% OF VN AT AGE 9	37.66796	41.68821	44.67787	47.14172	49.27787
% OF VN AT AGE 10	34.31841	38.14877	40.88352	43.16233	45.15715
% OF VN AT AGE 11	31.6225	34.86226	37.33083	39.40808	41.24204
% OF VN AT AGE 12	28.96145	31.81244	34.00712	35.87011	37.52751
% OF VN AT AGE 13	26.52095	28.98705	30.903	32.54223	34.01061
% OF VN AT AGE 14	24.29017	26.37696	28.01178	29.42044	30.69017
% OF VN AT AGE 15	22.26103	23.97555	25.32891	26.50258	27.56651
% OF VN AT AGE 16	20.42776	21.77832	22.85173	23.78809	24.64128
% OF VN AT AGE 17	18.7865	19.78257	20.57921	21.27784	21.91741
% OF VN AT AGE 18	17.33505	17.98719	18.51172	18.97397	19.39896
% OF VN AT AGE 19	16.07269	16.39247	16.65101	16.87985	17.09107
% OF VN AT AGE 20	15	15	15	15	15

.7	.8	.9	1	1.1	1.2
100	100	100	100	100	100
92.90902	93.70006	94.31451	94.80341	95.20002	95.52697
86.66775	87.86725	88.85188	89.66994	90.35941	90.94622
80.83617	82.3048	83.54456	84.60402	85.51772	86.31128
75.3129	76.95799	78.37588	79.61035	80.69297	81.64758
70.05202	71.80411	73.33855	74.69401	75.89862	76.9739
65.02441	66.82868	68.42943	69.8604	71.14623	72.3059
60.21115	62.0229	63.64791	65.11533	66.44643	67.65765
55.59944	57.38156	58.99509	60.46498	61.80937	63.04232
51.18072	52.90204	54.47339	55.916	57.24513	58.47266
46.94947	48.58358	50.0863	51.47544	52.7638	53.9612
42.90249	44.42703	45.83824	47.15094	48.37568	49.52052
39.03843	40.43451	41.7345	42.95061	44.09141	45.16341
35.35756	36.60928	37.78117	38.88312	39.922	40.90296
31.86147	32.93566	33.98516	34.9578	35.87899	36.75269
28.55299	29.47896	30.35413	31.18459	31.97443	32.72667
25.4361	26.18541	26.89658	27.57413	28.22106	28.83955
22.51579	23.08218	23.6218	24.13781	24.6323	25.10671
19.79813	20.17736	20.53994	20.88784	21.22234	21.54432
17.29018	17.47997	17.66202	17.83725	18.00625	18.16941
15	15	15	15	15	15

XIV. APPENDIX D. MODEL APPLICATION OF GROUP PROPERTY TO FIND VX

```
10    REM -------------------------------------------------------------
20    REM   Y  MODEL APPLICATION OF GROUP PROPERTY TO FIND VX
30    REM -------------------------------------------------------------
40    REM
50    REM  ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
60    REM  THE PROGRAM USED BASIC COMPUTER PROGRAMMING LANGUAGE
70    REM
80    REM   DEFINITION OF SYMBOLS
90    REM         VN=VALUE WHEN NEW      VX=VALUE WHEN AGE X
100    REM          I=DISCOUNT RATE        S=SALVAGE RATE
110    REM          N=AVERAGE LIFE         PL=PROBABLE LIFE
120    REM          ST=START OF LOOP       EN=END OF LOOP
130   REM         SP=STEP OF LOOP        FREQ=FREQUENCE
140   REM         K=SLOPE FACTOR
150   REM  ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
160   PRINT
170   PRINT " Y MODEL APPL. OF GROUP (WEIGHTING COND% AND PL) "
180   PRINT
190   PRINT " R3-10 IOWA TYPE SURVIVAL CURVE FOR FREQ.  AND PL"
200   PRINT
210   READ VN, S
220   LET I=.07
230   LET Q=1+I
240   LET V=VN*(1-S)
250   READ ST, EN, SP
260   DATA  100,.1
270   DATA .2,1.2,.1
280 FOR K=ST TO EN STEP SP
290   PRINT
300   PRINT "  K VALUE ";K
310   PRINT
320   FOR X=0 TO 17
330   GRANSUM=0
340   FOR N=1 TO 17
350   SUM1=0
360 FOR M=1 TO N
370    LET C1=(1-(.7*(M-1)/N)^K)*(1/Q)^M
380    SUM1=SUM1+C1
390 NEXT M
400   SUM2=0
410 FOR L=1 TO (N-X)
420    LET C2=(1-(.7*(L+X-1)/N)^K)*(1/Q)^L
430    SUM2=SUM2+C2
440 NEXT L
450   LET C=SUM2/SUM1
460 GOSUB 570
470   GRANSUM=C*FREQ+GRANSUM
```

```
480 NEXT N
490  GOSUB 760
500  LET A=VN*(1-S*(1/Q)^PL)*GRANSUM/100+VN*S*(1/Q)^(PL-X)
510  FRINT
520  PRINT "AGE"X,"CONDITION %",GRANSUM,"VALUE",A
530 NEXT X
540  PRINT ".............................................."
550 NEXT K
560  END
570 REM  SUBROUTINE FOR FREQUENCY OF RETIREMENT OF R3-10
580      IF N=1  THEN LET FREQ=. 23
590      IF N=2  THEN LET FREQ=. 48
600      IF N=3  THEN LET FREQ=. 88
610      IF N=4  THEN LET FREQ=1. 52
620      IF N=5  THEN LET FREQ=2. 43
630      IF N=6  THEN LET FREQ=3. 67
640      IF N=7  THEN LET FREQ=5. 3
650      IF N=8  THEN LET FREQ=7. 47
660      IF N=9  THEN LET FREQ=10. 18
670      IF N=10 THEN LET FREQ=13. 1
680      IF N=11 THEN LET FREQ=15. 13
690      IF N=12 THEN LET FREQ=14. 83
700      IF N=13 THEN LET FREQ=11. 86
710      IF N=14 THEN LET FREQ=7. 59
720      IF N=15 THEN LET FREQ=3. 87
730      IF N=16 THEN LET FREQ=1. 33
740      IF N=17 THEN LET FREQ=. 13
750  RETURN
760  REM  SUBROUTINE FOR PROBABLE LIFE OF GROUP OF R3-10
770      IF X>=0  AND X<=5  THEN LET PL=10
780      IF X>=6  AND X<=8  THEN LET PL=11
790      IF X>=9  AND X<=10 THEN LET PL=12
800      IF X>=11 AND X<=12 THEN LET PL=13
810      IF X>=13 AND X<=13 THEN LET PL=14
820      IF X>=14 AND X<=15 THEN LET PL=15
830      IF X>=16 AND X<=17 THEN LET PL=X
840  RETURN
```

XV. APPENDIX E. COMPARISONS OF VALUES

	Gas Tractor			Diesel Tractor			S.P combine		
	M.V.	T= 1.15	K= .85	M.V.	T= 1.15	K= .9	M.V.	T= 1.05	K= .5
AGE 0	100	100	100	100	100	100	100	100	100
AGE 1	84.3	93.17318	91.58513	86.3	94.60001	[illegible]	68.5	[illegible]	[illegible]
AGE 2	77.6	86.15871	83.72039	79.4	89.01225	86.71487	62.0	75.41279	[illegible]
AGE 3	71.4	78.98779	76.21933	73.0	83.25253	80.44775	[illegible]	68.24599	[illegible]
AGE 4	65.7	71.69922	69.03814	67.2	77.3417	74.3846	50.8	[illegible]	[illegible]
AGE 5	60.4	64.34179	62.15717	61.8	71.30665	68.51688	45.9	[illegible]	[illegible]
AGE 6	55.6	56.97845	55.56753	56.8	65.18288	62.84414	41.6	[illegible]	[illegible]
AGE 7	51.1	49.68405	49.26665	52.3	59.01314	[illegible]	[illegible]	[illegible]	[illegible]
AGE 8	47.0	42.55058	43.25608	48.1	52.85293	[illegible]	[illegible]	27.53757	[illegible]
AGE 9		35.69132	37.5406		46.76827	[illegible]	30.8	[illegible]	[illegible]
AGE 10	.	29.2407	32.12755	.	40.84007	42.16175	27.9	[illegible]	[illegible]
AGE 11		23.361	27.02644		35.1677	[illegible]		15.82129	18.12104
AGE 12		18.24509	22.24681		29.869	[illegible]		15	15
AGE 13		14.12313	17.80807		25.08602	[illegible]			
AGE 14		11.26767	13.71945		20.98753	25.04982			
AGE 15		10	10		17.77463	21.41037			
AGE 16					15.68531	18.05551			
AGE 17					15	15			

	Corn Picker			Hay Baler			F. Harvester		
	M.V.	T= .9	K= .3	M.V.	T= 1.00	K= .45	M.V.	T= 1.00	K= .5
AGE 0	100	100	100	100	100	100	100	100	100
AGE 1	66.8	82.52625	75.78031	67.4	87.68271	83.7099	65.9	[illegible]	[illegible]
AGE 2	58.8	67.5868	63.92437	61.0	76.13451	72.76336	61.4	72.887	[illegible]
AGE 3	51.7	54.98353	54.48818	55.6	65.38078	63.42391	54.6	61.12834	[illegible]
AGE 4	45.5	44.54163	46.63031	49.9	55.44355	55.20361	48.6	50.62517	[illegible]
AGE 5	40.0	36.10807	39.98438	45.1	46.4803	47.87708	43.2	41.45522	[illegible]
AGE 6	35.2	29.5493	34.3486	40.8	38.46108	41.31573	[illegible]	33.83963	[illegible]
AGE 7	31.0	24.75043	29.59922	36.9	31.44036	35.43892	[illegible]	[illegible]	[illegible]
AGE 8	27.2	21.61254	25.55659	33.4	25.62039	30.19278	[illegible]	23.25811	[illegible]
AGE 9	23.9	20.05217	22.4681	30.2	20.94292	25.5454	25.1	20.38032	23.31655
AGE 10	21.0	20	20		17.54278	21.4713	20.7	20	20
AGE 11					15.53976	17.9592			
AGE 12					15	15			
AGE 13									
AGE 14									
AGE 15									

	35 ton End dump truck			50 ton End dump truck			D9 dozer		
	M.V.	T= 1.00	K= .4	M.V.	T= 1.00	K= .4	M.V.	T= 1.05	K= .45
AGE 0	100	100	100	100	100	100	100	100	100
AGE 1	64.8	84.90704	78.91741	64.8	84.90704	78.91741	78.0	86.90542	81.41954
AGE 2	54.6	71.0036	66.05245	54.6	71.0036	66.05245	66.0	74.49009	69.26886
AGE 3	47.2	58.34982	55.38406	47.2	58.34982	55.38406	60.0	62.88355	59.09727
AGE 4	43.8	46.98295	46.22103	43.8	46.98295	46.22103	50.4	52.22013	50.32742
AGE 5	41.8	36.98249	38.25111	41.8	36.98249	38.25111	42.0	42.66783	42.69535
AGE 6	32.4	28.57767	31.29899	32.4	28.57767	31.29899	36.0	34.38726	36.05276
AGE 7	26.4	21.73604	25.25625	26.4	21.73604	25.25625	33.6	27.56488	30.30892
AGE 8	24.0	16.56405	20.05292	24.0	16.56405	20.05292	30.0	22.40763	25.40661
AGE 9	18.0	13.29371	15.64364	18.0	13.29371	15.64364	24.0	19.136	21.31036
AGE 10	12.2	12	12	13.2	12	12	18.0	18	18
AGE 11									
AGE 12									

	D9 C dozer			10 M Gal Water Wagon			CAT 637 Scraper		
	M.V	T= 1.05	K= .45	M.V.	T= .95	K= .45	M.V.	T= .95	K= .45
AGE 0	100	100	100	100	100	100	100	100	100
AGE 1	78.0	86.90542	81.41954	90.0	83.32754	80.56073	78.0	94.09648	81.41954
AGE 2	66.0	74.49009	69.26886	67.8	68.43766	67.75065	69.6	69.73357	69.26886
AGE 3	60.0	62.88355	59.09727	52.2	55.30807	56.96851	62.4	57.49077	59.09727
AGE 4	50.4	52.22013	50.32742	37.8	43.92138	47.61634	46.8	46.7325	50.32742
AGE 5	42.0	42.66783	42.69535	36.0	34.26663	39.41997	38.4	37.7094	42.69535
AGE 6	36.0	34.38726	36.05276	27.0	26.33847	32.22476	31.2	30.29767	36.05276
AGE 7	33.6	27.56488	30.30892	24.0	20.13779	25.8354	[illegible]	[illegible]	30.30892
AGE 8	30.0	22.40763	25.40661	20.4	15.67211	20.49112	[illegible]	20.74413	25.40661
AGE 9	24.0	19.136	21.31036	16.8	12.9529	15.85356	21.6	15.50392	21.31036
AGE 10	18.0	18	18	13.2	12	12	18.0	18	18
AGE 11									
AGE 12									

CAT 666 Scraper

	M.V.	T= .8	K= .3
AGE 0	100	100	100
AGE 1	68.4	76.49202	74.36699
AGE 2	56.4	58.17402	61.63523
AGE 3	44.4	44.04219	51.40906
AGE 4	34.8	33.29593	42.80725
AGE 5	25.2	25.29722	35.44507
AGE 6	18.0	19.53847	29.10969
AGE 7	15.6	15.61647	23.66979
AGE 8	14.4	13.21183	19.03974
AGE 9	13.2	12.07238	15.16242
AGE 10	12.0	12	12

CAT 825 Compactor

	M.V.	T= 1.1	K= 5
AGE 0	100	100	100
AGE 1	75.6	87.94824	82.73744
AGE 2	64.8	76.24735	70.68575
AGE 3	58.8	65.04388	60.44944
AGE 4	54.0	54.50159	51.53684
AGE 5	45.6	44.81289	43.72041
AGE 6	42.0	36.19454	36.87221
AGE 7	37.8	28.90025	30.91464
AGE 8	32.4	23.21299	25.7999
AGE 9	22.8	19.45731	21.49977
AGE 10	18.0	18	18

900 Portable Air Compressor

	M.V.	T= 1.15	K= 55
AGE 0	100	100	100
AGE 1	78.[illegible]	81.60299	78.5278
AGE 2	66.0	64.21135	[illegible].75067
AGE 3	50.6	48.24225	47.79569
AGE 4	36.0	34.1915	35.68949
AGE 5	24.0	22.64604	25.4114
AGE 6	16.[illegible]	14.[illegible]	14.3435
AGE 7	10.8	10	10

6" Rotary Drill

	M.V.	T= .9	K= .32
AGE 0	100	100	100
AGE 1	78.0	83.91553	79.05345
AGE 2	66.0	69.92247	67.5976
AGE 3	56.4	57.845	58.75629
AGE 4	46.8	47.52788	50.92195
AGE 5	42.0	38.83385	44.082
AGE 6	36.0	31.64186	38.06167
AGE 7	32.0	25.84613	32.75182
AGE 8	28.8	21.3542	28.08018
AGE 9	26.4	18.08665	23.99754
AGE 10	24.0	15.97518	20.47023
AGE 11		14.96223	17.47574
AGE 12		15	15
AGE 13			
AGE 14			
AGE 15			

35 Ton Motor Crane

	M.V.	T= 1.15	K= .8
AGE 0	100	100	100
AGE 1	92.8	88.19544	86.81624
AGE 2	70.8	76.49002	75.11833
AGE 3	62.4	65.03012	64.38147
AGE 4	54.6	53.99333	54.49717
AGE 5	44.4	43.59223	45.41951
AGE 6	38.4	34.08351	37.13104
AGE 7	33.6	25.77159	29.63072
AGE 8	30.0	19.02011	22.92887
AGE 9	18.0	14.26033	17.04331
AGE 10	12.0	12	12

150 Ton Crawler Crane

	M.V.	T= 95	K= 45
AGE 0	100	100	100
AGE 1	94.[illegible]	87.[illegible]	[illegible]
AGE 2	[illegible]	76.81[illegible]	76.[illegible]
AGE 3	[illegible]	68.90241	[illegible]
AGE 4	63.0	57.27[illegible]	[illegible]
AGE 5	55.2	48.81[illegible]	[illegible]
AGE 6	48.0	41.2[illegible]	[illegible]
AGE 7	44.[illegible]	[illegible]	[illegible]
AGE 8	[illegible]	[illegible]	[illegible]
AGE 9	[illegible]	[illegible]	[illegible]
AGE 10	[illegible]	[illegible]	[illegible]
AGE 11	31.2	[illegible]	[illegible]
AGE 12		[illegible]	[illegible]
AGE 13		[illegible]	[illegible]
AGE 14		10.15513	12.744[illegible]
AGE 15		10	10

90 KW Diesel Generator

	M.V.	T= 1.15	K= .8
AGE 0	100	100	100
AGE 1	84.0	93.69318	91.78594
AGE 2	75.6	87.22413	84.30719
AGE 3	72.0	80.52435	77.27632
AGE 4	66.0	73.93238	70.62758
AGE 5	63.6	67.19672	64.33018
AGE 6	60.0	60.47953	58.3684
AGE 7	56.4	53.95452	52.73511
AGE 8	52.8	47.41205	47.42869
AGE 9	46.8	41.26306	42.45164
AGE 10	42.0	35.53902	37.80967
AGE 11		30.3983	33.51119
AGE 12		26.02891	29.56697
AGE 13		22.65496	25.98998
AGE 14		20.54119	22.79529
AGE 15		20	20

D8 dozer

M.V.	T= .95	K= .45
100	100	100
82.2	88.3023	86.53278
73.8	77.52779	77.27846
65.4	67.68358	69.25864
56.4	58.66041	62.08874
51.0	50.53112	55.58471
44.4	43.25158	49.64124
42.0	36.81017	44.19088
39.0	31.19864	39.18805
33.6	26.41048	34.80119
28.8	22.44398	30.40832
	19.29524	26.59454
	16.97016	23.15031
	15.47344	20.0704
	14.81281	17.35317
	15	15

Industrial Forklift

M.V.	T= 1.[illegible]	K= [illegible]
100	100	100
81.9	[illegible]	[illegible]
75.[illegible]	[illegible]	[illegible]
65.0	62.20472	62.98603
54.1	52.90155	54.65241
40.7	44.25068	47.50153
35.8	37.12834	41.3061
33.3	[illegible]	[illegible]
31.2	[illegible]	[illegible]
27.0	25.29694	27.85862
23.7	25	25

Pickup Truck

	M.V.	T= .9	K= .3
AGE 0	100	100	100
AGE 1	80.4	83.22904	77.47805
AGE 2	67.3	68.47116	65.66785
AGE 3	54.4	55.54902	55.87133
AGE 4	46.2	44.30391	47.34588
AGE 5	35.5	34.59504	39.7637
AGE 6	28.4	26.29788	32.94031
AGE 7	22.5	19.30171	26.75891
AGE 8		13.51011	21.1404
AGE 9		8.837207	16.02904
AGE 10		[illegible]	11.38463
AGE 11		2.562229	7.177956
AGE 12		.8413285	3.388006
AGE 13		0	0